Relativized Exhaustivity

Mention-some and Uniqueness

Yimei Xiang
yimei.xiang@rutgers.edu
Rutgers University

Logic Colloquium, UConn, March 2022

Mention-some

- *Wh*-questions with an existential goal-oriented modal (abbr. '*can*-questions') can be naturally addressed by **mention-some (MS) answers** (Gr&S 1984). They also admit **mention-all (MA)** answers, stated either as conjunctions or as free-choice disjunctions.

(1) (A's belief: There are two coffee places nearby, namely Starbucks and Peet's.)
Q: 'Where can we get coffee?' A: ...

a. 'Starbucks.\' (MS)
b. 'Starbucks and Peet's.\' (conjunctive MA)
c. 'Starbucks or Peet's.\' (disjunctive MA)

⇒ Question interpretations are not always exhaustive.

Uniqueness

- Questions with a singular *which*-phrase (abbr. 'singular *wh*-questions') are subject to a uniqueness effect.

(2) Which child came? $\rightsquigarrow$ *Only ONE of the children came.*

⇐ Dayal (1996): Question interpretations must be exhaustive; a question is defined only if it has an exhaustive true answer.

Dilemma! ☞ **Solution: Relativized Exhaustivity**

Introduction

Part I: Composing *can*-questions

Part II: Solving the dilemma between uniqueness and MS.

This talk is based on Xiang, Y. (2021) 'Relativized Exhaustivity: Mention-some and Uniqueness' (*ling.auf.net/lingbuzz/005322*), with lots of cuts and simplifications.

Relativized Exhaustivity: Mention-Some and Uniqueness
Yimei Xiang
October 2021

Wh-questions with the modal verb *can* admit both mention-some (MS) and mention-all (MA) answers. This paper argues that we should treat MS as a grammatical phenomenon, primarily determined by the grammar of a *wh*-interrogative. I assume that MS and MA answers can be modeled using the definition of answerhood (Fox 2013) and attribute the MS/MA ambiguity to structural variations within the question nucleus. The variations are: (i) the scope ambiguity of the higher-order *wh*-trace, and (ii) the absence/presence of an anti-exhaustification operator. However, treating MS answers as complete answers in this way contradicts the widely adopted analysis of uniqueness effects in questions of Dayal 1996, according to which the uniqueness effects of singular *which*-phrases come from an exhaustivity presupposition, namely that a question must have a unique exhaustive true answer. To solve this dilemma, I propose that question interpretations presuppose 'Relativized Exhaustivity': roughly, the exhaustivity in questions is evaluated relative to the accessible worlds as opposed to the anchor/utterance world. Relativized Exhaustivity preserves the merits of Dayal's exhaustivity presupposition while permitting MS; moreover, it explains the local-uniqueness effects in modalized singular *wh*-questions.

Format:	[pdf]
Reference:	lingbuzz/005322 (please use that when you cite this article)

Part I: Composing *can*-questions

- Earlier approaches account for the licensing/distribution of MS interpretations by **pragmatic factors**, such as the conversational goals of the questioner. (Gr&S 1984; Ginzburg 1995; van Rooij 2003; a.o.)

(3) Where can we get coffee?

  a. MS: to get a cup of coffee.
  b. MA: to investigate the local coffee market.

- However, MS answers to *can*-questions are subject to a grammatical constraint, called '**mention-one-only**', which cannot be explained by pragmatics.

☞ I analyze MS as a **grammatical** phenomenon (cf. George 2011; Fox 2013). Its primary origin is the existential modal verb *can* inside the question nucleus.

The 'mention-one-only' constraint

The 'mention-one-only' constraint

A MS answer to a *can*-question specifies **only one option** that resolves the question.

☞ 'mention-some' = 'mention-one'

Two contrasts between **mention-one (MO)** and **mention-few (MF)**:

1. Embeddings of *can*-questions allow for MO and MA interpretations but not non-exhaustive MF interpretations, even in cases where MF fits with the conversational goal the best.

   (4) Alice: 'We are looking for a campus venue for the upcoming conference. We need to identify three options. Do you know where on campus one can hold a large conference?'

   Bob:

   a. 'Yes, but I only know one such place.'
   b. #'No/Sorry, I only know one such place.'

   Despite the conversational goal being 'mention-three', Bob cannot felicitously reply with a denial while admitting that he knows one possible venue.

2. Matrix *can*-questions in discourse ...

The ‘mention-one-only’ constraint (cont.)

2. In discourse, unlike MO answers, MF answers are read exhaustively, unless ignorance is marked separately.

(5) (Alice has an electric slicer, which comes with 10 blades. These blades have different colors and shapes, designed for different ingredients. Now, while Alice is cooking, her friend Bob comes to help her cut carrots.)

Bob: ‘Which blade can I use to cut carrots?’
Alice:

a. ‘The green one.’
$\not\rightsquigarrow$ *But not the other blades.* (MO: likely non-exh)

b. ‘The green one or the black one.’
$\rightsquigarrow$ *But not the other blades.* (MF: likely exh)

The ‘mention-one-only’ constraint (cont.)

- In comparison: for answers to *ex*-questions (viz., questions with a partiality marker), there is no contrast in exhaustivity between MO and MF.

(6) Who is on your committee, for example?

a. Andy is on my committee.\ (MO: non-exh)
b. Andy and Billy are on my committee.\ (MF: non-exh)

One can make an exact ‘mention-*N*’ inquiry by saying “Give me *N* examples”.

☞ The source of non-exhaustivity in *can*-questions differs from that in *ex*-questions.

☞ Next: Propose a compositional analysis of *can*-questions and account for the ‘mention-one-only’ constraint.

Answerhood

Truth and **exhaustivity** are encoded within the **answerhood operations**.

- Fox (2013): A true answer is complete[1] iff it isn't asymmetrically entailed by any of the true answers, called maximally **(max-)informative true** answers.[2]

(7) $\textsc{Ans}(w)(Q) = \{p \mid w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow q \not\subset p]\}$

- Max-informativity is weaker than exhaustivity. It derives either MS or MA, depending on the logical relation of the propositions in the answer space Q.

(8) Let ϕ_a and ϕ_b be mutually independent and true in w:

a. $Q_1 = \{\phi_a, \phi_b\}$ $\quad$ $\textsc{Ans}(w)(Q_1) = \{\phi_a, \phi_b\}$ $\quad$ (MS)

b. $Q_2 = \{\phi_a, \phi_b, \phi_a \wedge \phi_b\}$ $\quad$ $\textsc{Ans}(w)(Q_2) = \{\phi_a \wedge \phi_b\}$ $\quad$ (MA)

☞ MS arises only if there is a world where Q has **multiple** max-informative true answers, unavailable if Q is closed under conjunction.

[1] An answer being 'complete' means that this answer is the expected type of direct answers.

[2] In the paper, for ease of modeling relativized exhaustivity, I define a *wh*-question as a 'topical property', namely, a function that maps an individual/GQ in the quantification domain of the *wh*-phrase (viz., a short answer) to a proposition in the answer space Q. This assumption is omitted in the slides.

Deriving first-order MS interpretations

MS answers are subject to **local exhaustivity** and **mutual independence**:

- (a)-vs-(b&c): a MS answer should specify all the members of a possible committee.
- (b) is a good MS answer, although (seemingly) (c) asymmetrically entails (b).

(9) (The committee can be formed in two ways: it should either have the two members Andy and Billy, or have the three members Andy, Billy, and Cindy.)

Who can serve on the committee?

a. #Andy can.\ $\Diamond O_C[\text{serve-on}(a)]$
b. Andy and Billy can.\ $\text{serve-on}(a \oplus b)$ $\Diamond O_C[\text{serve-on}(a \oplus b)]$
c. Andy, Billy, and Cindy can.\ $\text{serve-on}(a \oplus b \oplus c)$ $\Diamond O_C[\text{serve-on}(a \oplus b \oplus c)]$

. . . .

Analysis: An exhaustification operator O ($\approx$ *only*) is applied to the local VP and is associated with the e-type *wh*-trace x.

(10) $[\![O_C]\!] = \lambda p \lambda w.p(w) = 1 \wedge \forall q \in C[p \not\subseteq q \rightarrow q(w) = 0]$ (Chierchia et al. 2012; a.o.)

(11) a. [who λx_e [**can** [O_C [x serve on the committee]]]]
b. $Q = \{\Diamond O_C[\text{serve-on}(x)] \mid x \in \text{hmn}_@\}$
c. $\textsc{Ans}(w)(Q) = \{\Diamond O_C[\text{serve-on}(a \oplus b)], \Diamond O_C[\text{serve-on}(a \oplus b \oplus c)]\}$

The local O-operator asserts local exhaustivity: (a) is false.
Its non-monotonicity makes the individual answers mutually independent: (c) $\not\Rightarrow$ (b).

Deriving higher-order MS interpretations

Analysis: The fronted *wh*-phrase binds a higher-order trace π (of type $\langle et, t\rangle$) between *can* and the local O-operator.

(12) Who can chair the committee?
[who $\lambda\pi_{\langle et,t\rangle}$ [**can** [π λx_e [O_C [$_{\phi_x}$ x chair the committee]]]]]

(Only Andy and Billy can chair the committee. Co-chairing is disallowed.)

$$\Diamond(O_C\phi_a \wedge O_C\phi_b)$$

$$\Diamond O_C\phi_a \qquad \vee \qquad \Diamond O_C\phi_b$$

$$\Diamond(O_C\phi_a \vee O_C\phi_b)$$

Conjunctive
(contradictory)

Individual
(independent)

Disjunctive
(partial)

- ✓ Explain the **MS-licensing** effect of *can*:
  With the presence of *can*, the independent answers are **not mutually exclusive**.
- ✓ Explain the '**mention-one-only**' constraint:
  - In reply to a *can*-question with a MS interpretation, only individual answers, which each specify exactly one option, are possibly max-informative.
  - If the addressee uses a Boolean coordination to specify multiple options, they must be understanding the question with a MA interpretation.

(13) Who can chair (the committee)?

a. Andy can. (MS)

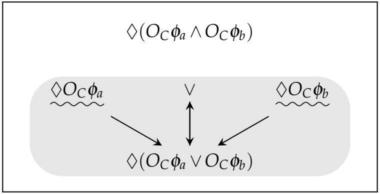
b. Andy can and Billy can. (Conjunctive MA)

Analysis: A conjunctive MA interpretation arises if the higher-order *wh*-trace π takes scope **above** the existential modal verb *can*.

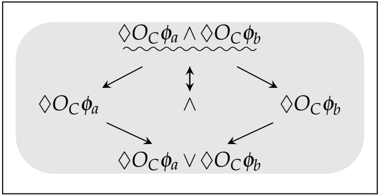
(14)

a. [who $\lambda\pi_{\langle et,t\rangle}$ [**can** [π λx_e [O_C [$_{\phi_x}$ x chair]]]]] MS

b. [who $\lambda\pi_{\langle et,t\rangle}$ [π λx_e [**can** [O_C [$_{\phi_x}$ x chair]]]]] Conjunctive MA



$\Diamond \gg \pi$: MS



$\pi \gg \Diamond$: conjunctive MA

The conjunctive answer isn't contradictory; it is logically equivalent to the conjunction of the two individual answers.

Deriving disjunctive MA interpretations

A disjunctive answer to a *can*-question is read with **ignorance** or **free choice (FC)**.

(15) 'Who can chair the committee?' 'Andy or Billy (can).'

a. $\rightsquigarrow$ *Either Andy or Billy can chair, but I don't know who.* (Ignorance: partial)

b. $\rightsquigarrow$ *Andy can chair, and Billy can chair, too.* (FC: mention-few/all)

☞ In a *can*-question, a disjunctive MA interpretation arises only if felicitous disjunctive answers can be understood as universal FC statements.

. . . .

Universal FC is derived by **anti-exhaustification** (a la Kratzer & Shimoyama 2002):

$$\left.\begin{array}{l} \textsc{dou}\Diamond[\phi \vee \psi] \\ \textsc{dou}[\Diamond\phi \vee \Diamond\psi] \\ \textsc{dou}[\Diamond\phi \wedge \Diamond\psi] \end{array}\right\} \Leftrightarrow \Diamond\phi \wedge \Diamond\psi$$

The anti-exhaustification operator

$$[\![\textsc{dou}_C]\!] = \lambda p \lambda w : p(w) = 1 \wedge \forall q \in \textsc{AntiExcl}(p, C)[O^{\text{IE}}_C(q)(w) = 0]$$
$$\text{where } \textsc{AntiExcl}(p, C) = (C - \textsc{IExcl}(p, C)) - \{p\}$$

(The prejacent is true, while for each anti-excludable alternative of the prejacent in C, its exhaustification is false.)

The anti-exhaustification operator is called 'DOU', in reference to the Mandarin particle *dou*, which displays the properties of such operator overtly in questions and declaratives.

(16) (**Dou**) shui keyi jiao jichu hanyu?
(DOU) who can teach Intro Chinese
Without *dou*: 'Who can teach Intro Chinese?' (✓MS, ✓MA)
With *dou*: 'Who all can teach Intro Chinese?' (✗MS, ✓MA)

(17) Yuehan huozhe Mali (**dou**) keyi jiao jichu hanyu.
John or Mary (DOU) can teach Intro Chinese
Without *dou*: 'Either J or M can teach IC.' (Ignorance)
With *dou*: 'J and M (and possibly others) can teach IC.' (Universal FC)

Anti-exhaustification and anti-excludable alternatives

I divide alternatives into two categories: **innocently (I-)excludable** alternatives participate in exhaustification (Fox 2007), while **anti-excludable** alternatives participate in anti-exhaustification.[3]

	Anti-excludable		
	Weaker	Neither	I-excludable
$\phi \wedge \psi$ $\Box\phi \wedge \Box\psi, \Box(\phi \wedge \psi)$ $\Diamond\phi \wedge \Diamond\psi, \Diamond(\phi \wedge \psi)$	ϕ, ψ $\Box\phi, \Box\psi$ $\Diamond\phi, \Diamond\psi$		
$\phi \vee \psi$ $\Diamond\phi \vee \Diamond\psi, \Diamond(\phi \vee \psi)$ $\Box\phi \vee \Box\psi$		ϕ, ψ $\Diamond\phi, \Diamond\psi$ $\Box\phi, \Box\psi$	
$\Box(\phi \vee \psi)$			$\Box\phi, \Box\psi$

(18) [DOU_C [$_S$ Andy or Billy can chair the committee]]

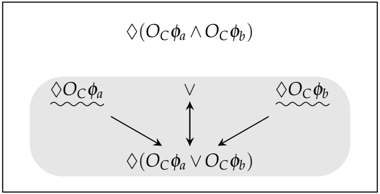
$$
\begin{aligned}
[\![\text{DOU}_C(\text{S})]\!] &\Leftrightarrow [\Diamond\phi_a \vee \Diamond\phi_b] \wedge \neg O^{\text{IE}}_C \Diamond\phi_a \wedge \neg O^{\text{IE}}_C \Diamond\phi_b \\
&\Leftrightarrow [\Diamond\phi_a \vee \Diamond\phi_b] \wedge [\Diamond\phi_a \rightarrow \Diamond\phi_b] \wedge [\Diamond\phi_b \rightarrow \Diamond\phi_a] \\
&\Leftrightarrow [\Diamond\phi_a \vee \Diamond\phi_b] \wedge [\Diamond\phi_a \leftrightarrow \Diamond\phi_b] \\
&\Leftrightarrow \Diamond\phi_a \wedge \Diamond\phi_b
\end{aligned}
$$

[3] I treat exhaustification and anti-exhaustification as two independent operations (contra Fox 2007; Bar-Lev & Fox 2020), so as to separate FC from exclusivity.

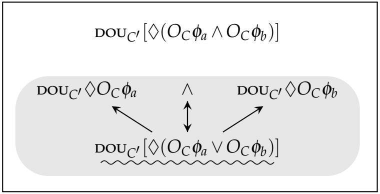
Deriving disjunctive MA interpretations (cont.)

Analysis: Apply the anti-exhaustification operator DOU within the question nucleus and let it be associated with the higher-order *wh*-trace π.[4]

(19) Who can chair (the committee)?
$[$ who $\lambda\pi_{\langle et,t\rangle}$ $[$ ($\textsc{dou}_{C'}$) $[$ can $[$ π λx_e $[$ O_C $[_{\phi_x}$ x chair $]]]]]]$



$\Diamond \gg \pi$ (without DOU): MS



$\Diamond \gg \pi$ (with DOU): disjunctive MA

DOU strengthens the disjunctive answer into a **universal FC** statement, making the answer space closed under conjunction.

[4] The scopal relation between π and *can* doesn't affect this derivation: $\textsc{dou}_C[\Diamond(\phi \vee \psi)] \Leftrightarrow \textsc{dou}_C[\Diamond\phi \vee \Diamond\psi]$.

Interim summary

- The 'mention-one-only' constraint argues that the distribution/licensing of MS isn't purely determined by pragmatics.
- The MS/MA ambiguity in *can*-questions comes from minimal structural variations within the question nucleus:
  1. the scopal relation between *can* and the higher-order *wh*-trace
  2. the presence/absence of the **anti-exhaustification** operator

- Other issues:
  - the distribution of MS interpretations in non-*can*-questions
  - the distribution of FC-disjunctive answers (related: modal obviation)
  - ...

Part II: A dilemma and Relativized Exhaustivity

- Dilemma: Allowing complete answers to be non-exhaustive conflicts with Dayal's **exhaustivity presupposition**, which is important for explaining the **uniqueness** effects of singular *which*-phrases.
- Solution: **Relativized Exhaustivity**. This condition has the effect of evaluating exhaustivity relative to the accessible worlds, as opposed to the utterance world.

Uniqueness of singular *wh*-questions

Singular *wh*-questions are subject to uniqueness.

(20) Which child came?
$\rightsquigarrow$ *Only ONE of the children came.*

Dayal 1996

1. **Exhaustivity presupposition (EP)**: A question is defined only if it has an exhaustive true answer, i.e., a true answer that entails all the true answers.
2. A plural *wh*-question has sum-based answers while a singular *wh*-question doesn't.

(21) (Among the children under consideration, only Andy and Billy came. The speaker knows that multiple children came, but she doesn't know who they are.)

a. Which children came?
$\{\lambda w.\text{came}_w(a), \lambda w.\text{came}_w(b), \lambda w.\text{came}_w(a \oplus b)\}$ (Dayal's EP is satisfied)

b. #Which child came?
$\{\lambda w.\text{came}_w(a), \lambda w.\text{came}_w(b)\}$ (Dayal's EP is violated)

On higher-order *wh*-quantification:

- Uniqueness effects also argue that singular *wh*-questions cannot have answers formed out of Boolean conjunctions like $a^{\Uparrow} \cap b^{\Uparrow}$:

(22) (Among the children under consideration, only Andy and Billy came.)
\# Which child came?

$$\left\{ \begin{array}{c} \cancel{\lambda w.\text{came}_w(a) \wedge \text{came}_w(b)} \\ \lambda w.\text{came}_w(a), \quad \lambda w.\text{came}_w(b) \\ \lambda w.\text{came}_w(a) \vee \text{came}_w(b) \end{array} \right\}$$

- This disjunction–conjunction asymmetry is also seen in modalized questions:

(23) Which textbook can I use for this class?

a. Book A or Book B.
b. #Book A and Book B.

- In sum:

	which-NP$_{singular}$	*which*-NP$_{plural}$
first-order	a, b	$a, b, a \oplus b$
higher-order	$a^{\Uparrow}, b^{\Uparrow}, a^{\Uparrow} \cup b^{\Uparrow}$	$a^{\Uparrow}, b^{\Uparrow}, ..., a^{\Uparrow} \cup b^{\Uparrow}, a^{\Uparrow} \cap b^{\Uparrow}, ...$

Consequences of abandoning Dayal's EP: over-predictions of MS

Without Dayal's EP, the current proposal over-predicts MS interpretations.

1. Questions with a **uniqueness** presupposition

(24) *Which child came?* (w: both a and b came.)
$Q_{\text{FO}} = \{\phi_a, \phi_b\}$; $Q_{\text{HO}} = \{\phi_a, \phi_b, \phi_a \vee \phi_b\}$

a. With Dayal's EP: MA with uniqueness ✓
b. Without Dayal's EP: MS without uniqueness ✗

2. *Wh*-questions with a **non-distributive predicate**: Q is closed under conjunction in the higher-order interpretation, but not in the first-order interpretation.

(25) *Which children formed a team?* (w: $a + b$ formed one, $c + d$ formed one.)
$Q_{\text{FO}} = \{\phi_{a\oplus b}, \phi_{c\oplus d}\}$; $Q_{\text{HO}} = \{\phi_{a\oplus b}, \phi_{c\oplus d}, \phi_{a\oplus b} \vee \phi_{c\oplus d}, \phi_{a\oplus b} \wedge \phi_{c\oplus d}\}$

a. With Dayal's EP: MA without uniqueness ✓
b. Without Dayal's EP: Ambiguous between MS and MA ✗

3. *Wh*-questions with an **existential indefinite** are predicted to behave the same as *can*-questions in licensing MS. ✗

(26) *Which movie(s) did one of the boys watch?*
'Which movie(s) y is/are s.t. one of the boys watched y?' (Individual)

a. (One of the boys watched) *Ironman* and (one watched) *Hulk*.
b. #(One of the boys watched) *Ironman*.

A dilemma

- Dayal's EP explains uniqueness, but it is **too strong** to allow for MS;
- Abandoning Dayal's EP makes MS interpretations possible, but the assumed answerhood is **too weak** to capture uniqueness and over-generates MS.

Existing solutions to the MS-vs-uniqueness dilemma:

- Fox 2018, 2020: partition-by-exhaustification
- Hirsch and Schwarz 2020: presuppositional *which*

(Reviewed in the paper: *ling.auf.net/lingbuzz/005322*)

A challenge to Dayal's EP: local uniqueness

Modalized singular *wh*-questions are subject to **local uniqueness**, regardless of the **modal flavor** or the **modal force**.

- Hirsch and Schwarz 2020 on singular $\Diamond$-questions:

(27) Which letter could we add to *fo_m* (to form a word)? (priority)
A or *r*. ('The unique letter that we add to *fo_m* could be *a* and could be *r*.')

(28) Which letter could be missing in *fo_m*? (epistemic)
A or *r*. ('The unique letter missing in *fo_m* could be *a* and could be *r*.')

- Xiang 2021 on singular $\Box$-questions:

(29) Which chapter do we have to assign (to the students)?
$\rightsquigarrow$ *We are not allowed to assign more than one chapter.*
Chapter 1 or chapter 2, either is good.

Dayal's EP cannot capture local uniqueness effects. This problem is independent of MS.

Proposal: Relativized Exhaustivity

Intuitively, uniqueness appears to be 'local' if exhaustivity is evaluated relative to the **accessible worlds**, as opposed to the utterance world.

(30) a. Which chapter can we assign?/ Which chapter do we have to assign?
b. $\forall w' \in M_w$ [we assign a chapter in w' $\rightarrow$ we assign only one chapter in w']

A modalized *wh*-question $\Diamond Q/\Box Q$ is defined only if the non-modalized counterpart Q satisfies Dayal's exhaustivity requirement in every accessible world where Q has a true answer.

Next: How can we capture this idea analytically?

In model-theoretic compositional semantics, it's technically difficult to extract Q from $\Diamond Q/\Box Q$, letting alone to evaluate the exhaustivity of Q.

Defining Relativized Exhaustivity

The idea of relativized exhaustivity can be captured by substituting the **modal base** — interpreting Q w.r.t. an accessible world is just like interpreting $\Diamond Q/\Box Q$ w.r.t. a modal base introducing a singleton set of accessible worlds.

Relativized Exhaustivity (RelExh)

Dayal's EP must be satisfied relative to every modal base that introduces a singleton set of accessible worlds which verifies a true answer.[5,6]

(31) $\forall M'_{\langle s,st\rangle}[|M'_w| = 1 \wedge M'_w \subseteq M_w \wedge \exists\alpha \in ([\![Q]\!]^M_w \cap [\![Q]\!]^{M'}_w) \rightarrow \mathbf{DEP}(w, M', [\![Q]\!])]$

a. $[\![Q]\!]^M_w := \{\alpha \mid \alpha \in \mathrm{Dom}([\![Q]\!]^M) \wedge [\![Q]\!]^M(\alpha)(w) = 1\}$

b. $\mathbf{DEP}(w, M, [\![Q]\!]) := \exists\alpha[\alpha \in [\![Q]\!]^M_w \wedge \forall\beta[\beta \in [\![Q]\!]^M_w \rightarrow [\![Q]\!]^M(\alpha) \subseteq [\![Q]\!]^M(\beta)]]$

---

[5] For a set of worlds W, W verifies $\Diamond\phi$ in w iff there is a w-accessible world w' in W s.t. $w' \in \phi$; W verifies $\Box\phi$ in w iff for every w-accessible world w' in W, $w' \in \phi$.

[6] $[\![Q]\!]$ is a function-like denotation that takes short answers as arguments. RelExh is defined based on short answers because the meaning of a short answer (cf. sentential answer) is modal independent. Another option is to define modalized sentences as functions from modal bases to propositions (see A4). This option allows RelExh to be defined based on sentential answers, since sentential denotations are arrived at modal independently.

Prediction I: permitting (first-order) MS

(32) Who can chair the committee?

$$\begin{tikzcd}[row sep=small] & w_1 \arrow[r, dotted, no head] & \Diamond O_C \phi_a \\ w \arrow[ur] \arrow[dr] & & \\ & w_2 \arrow[r, dotted, no head] & \Diamond O_C \phi_b \end{tikzcd}$$

Assume a multiple-choice scenario:

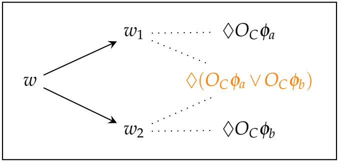
$$\text{Let } \begin{pmatrix} M_w = \{w_1, w_2\} \\ M_w^1 = \{w_1\} \\ M_w^2 = \{w_2\} \end{pmatrix} \text{ and chair-t.c.} = \begin{bmatrix} w_1 \to \{a\} \\ w_2 \to \{b\} \\ \ldots \end{bmatrix} \text{; then:}$$

a. $[\![\mathrm{Q}]\!]_w^M = \{a, b\}$ ETA: non-existent
b. $[\![\mathrm{Q}]\!]_w^{M^1} = \{a\}$ ETA: $\Diamond O_C \phi_a$
c. $[\![\mathrm{Q}]\!]_w^{M^2} = \{b\}$ ETA: $\Diamond O_C \phi_b$

- Given M, the question doesn't have an exhaustive true answer (ETA).
- However, if the question is interpreted relative to M^1, $\Diamond O_C \phi_a$ would be the ETA in w; likewise for M^2. Hence, given M, **RelExh is satisfied** in w. ✓

Prediction I: permitting (higher-order) MS

(33) Who can chair the committee?



Assume a multiple-choice scenario:

$$\text{Let} \begin{pmatrix} M_w = \{w_1, w_2\} \\ M_w^1 = \{w_1\} \\ M_w^2 = \{w_2\} \end{pmatrix} \text{ and chair-t.c.} = \begin{bmatrix} w_1 \to \{a\} \\ w_2 \to \{b\} \\ \ldots \end{bmatrix} \text{; then:}$$

a. $[\![\mathrm{Q}]\!]_w^M = \{a^{\Uparrow}, b^{\Uparrow}, a^{\Uparrow} \cup b^{\Uparrow}\}$ ETA: non-existent

b. $[\![\mathrm{Q}]\!]_w^{M^1} = \{a^{\Uparrow}, a^{\Uparrow} \cup b^{\Uparrow}\}$ ETA: $\Diamond O_C \phi_a$

c. $[\![\mathrm{Q}]\!]_w^{M^2} = \{b^{\Uparrow}, a^{\Uparrow} \cup b^{\Uparrow}\}$ ETA: $\Diamond O_C \phi_b$

Generalization of RelExh to MS interpretations

The MS interpretations of ⌜*Wh-A can P?*⌝ satisfy RelExh iff
the interpretations of ⌜*Wh-A P?*⌝ satisfy Dayal's EP in every accessible world where it has a true answer.

Prediction II: avoiding over-predictions of MS

- On *wh*-questions with a **non-distributive predicate**:

(34) Which children formed a team?

In a multiple-team scenario, the first-order interpretation of (34) violates RelExh, just as it violates Dayal's EP.

- On *wh*-questions with **indefinites**:

(35) Which movie(s) did one of the boys watch?

RelExh makes exhaustivity be evaluated relative to a smaller modal base, not relative to a smaller discourse domain. The essence of RelExh is that language users naturally shift the origo for interpretation to worlds (or situations) where the state under discussion actually emerges. In contrast, the discourse domain is independent of perspective. Hence, for any non-modalized questions, including questions with indefinites, RelExh makes the same prediction as Dayal's EP.

Prediction III-a: regarding uniqueness

For non-modalized questions

- The selection of modal base doesn't affect non-modalized questions. Thus, RelExh carries forward the merits of Dayal's EP in explaining their uniqueness effects.

(36) Which child came?
$\rightsquigarrow$ *Only ONE of the children came.*

For modalized questions

- Let's focus on local uniqueness:

(37) Which chapter do we have to assign?
$\rightsquigarrow$ *We shouldn't assign more than one chapter.* (Universal)

(38) Which chapter can we assign?
$\rightsquigarrow$ *We can assign one chapter, but not more.* (Universal)

- Each of these questions also admits an interpretation that presupposes global uniqueness: *There is only one chapter that we have to/ can assign.*

Prediction III-b: regarding uniqueness in *have to*-questions

(39) Which chapter do we have to assign? (higher-order, with $\Box \gg \pi$)
$\rightsquigarrow$ *We shouldn't assign more than one chapter.*

$$\begin{tikzcd}
& \Box(\phi_{c_1} \wedge \phi_{c_2}) \arrow[dl] \arrow[dr] & \\
\Box\phi_{c_1} \arrow[dr] & \vee \arrow[d] & \Box\phi_{c_2} \arrow[dl] \\
& \Box(\phi_{c_1} \vee \phi_{c_2}) &
\end{tikzcd}$$

Assume a local-uniqueness-violating scenario:

$$\text{Let } \left(\begin{array}{l} M_w = \{w_1, w_2, w_3\} \\ M'_w = \{w_3\} \end{array} \right), \text{ assign} = \left[\begin{array}{l} w_1 \to \{c_1\} \\ w_2 \to \{c_2\} \\ w_3 \to \{c_1, c_2\} \\ \ldots \end{array} \right]; \text{ then:}$$

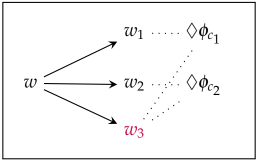
a. $[\![\mathrm{Q}]\!]_w^M = \{c_1^{\Uparrow} \cup c_2^{\Uparrow}\}$ ETA: $\Box(\phi_{c_1} \vee \phi_{c_2})$
b. $[\![\mathrm{Q}]\!]_w^{M'} = \{c_1^{\Uparrow}, c_2^{\Uparrow}, c_1^{\Uparrow} \cup c_2^{\Uparrow}\}$ ETA: non-existent

- With M, the question has a unique true answer in w; thus, **Dayal's EP is satisfied**. ✗
- $\{w_3\}$ verifies the true answer $\Box(\phi_{c_1} \vee \phi_{c_2})$. However, given any M' s.t. $M'_w = \{w_3\}$, the question has multiple true answers but no ETA in w. **RelExh is violated**. ✓

☞ RelExh is a **mandatory** condition for question interpretation. If it were optional, or if it only came into play in cases where Dayal's EP is violated, we would expect local uniqueness to be optional or absent in singular $\Box$-questions.

Prediction III-c: regarding uniqueness in *can*-questions

(40) Which chapter can we assign? (first-order MS)
$\rightsquigarrow$ *We can assign exactly one chapter, but not more.*



Assume the same local-uniqueness-violating scenario:

$$\text{Let} \begin{pmatrix} M_w = \{w_1, w_2, w_3\} \\ M'_w = \{w_3\} \end{pmatrix}, \text{assign} = \begin{bmatrix} w_1 \rightarrow \{c_1\} \\ w_2 \rightarrow \{c_2\} \\ w_3 \rightarrow \{c_1, c_2\} \\ \ldots \end{bmatrix}; \text{then:}$$

a. $[\![\mathrm{Q}]\!]_w^M = \{c_1, c_2\}$ ETA: non-existent
b. $[\![\mathrm{Q}]\!]_w^{M'} = \{c_1, c_2\}$ ETA: non-existent

- The true answers $\Diamond\phi_{c_1}$ and $\Diamond\phi_{c_2}$ can be verified by $\{w_3\}$. Given any M' s.t. $M'_w = \{w_3\}$, the question has no ETA in w. Hence, **RelExh is violated**. ✓
- (Same idea applies to the higher-order MS interpretation.)

Conclusions

- **Observation**: MS answers are subject to a 'mention-one-only' constraint, which cannot be explained by pragmatics.
- **Analysis**: MS and MA answers are derived as complete answers by a single answerhood operation; the MS/MA ambiguity in *can*-questions comes from minimal structural variations within the question nucleus.
- **Dilemma**: Allowing non-exhaustive answers to be complete would cause a troubling conflict with Dayal's EP. This condition is crucial in accounting for the uniqueness effects in questions.
- **Solution**: Question interpretations can violate Dayal's EP but mandatorily presuppose 'Relativized Exhaustivity'. This condition permits MS where needed, without over-generating it. Moreover, it explains the local-uniqueness effects in modalized singular *wh*-questions.

Thank you!

(For references, please see the paper.)

A1: Modal obviation of FC: Relativized Exclusivity

Modal obviation: Universal FC is only possible in $\Diamond$-sentences.

Relativized Exclusivity (RelExcl) (cf. the viability constraint of Dayal 2013)

For every anti-excludable alternative ϕ stronger than the prejacent, every minimal set of accessible worlds that verifies ϕ also verifies the (IE-based) exhaustification of ϕ.

(41) $[\![\textsc{dou}_C(\text{S})]\!]^M$ is defined in w only if
$\forall\phi[[\![\phi]\!]^M \in \textsc{AntiExcl}([\![\text{S}]\!]^M, C) \wedge [\![\phi]\!]^M \subset [\![\text{S}]\!]^M$
$\rightarrow \forall M'_{\langle s,st\rangle}[M'_w \text{ is a minimal subset of } M_w \text{ s.t. } [\![\phi]\!]^{M'}(w) = 1 \rightarrow O^{\text{IE}}_{C'}([\![\phi]\!]^{M'})(w) = 1]]$

(42) *$\textsc{dou}_C$ [John or Mary teach Intro Chinese]
$\textsc{dou}_C[\phi_j \vee \phi_m]$ is defined only if $O\phi_j \wedge O\phi_m$, which is a contradiction.

(43) $\textsc{dou}_C$ [John or Mary can teach Intro Chinese]

a. $\textsc{dou}_C[\Diamond(\phi_j \vee \phi_m)]$ is defined only if $\Diamond O\phi_j \wedge \Diamond O\phi_m \wedge \neg\Diamond(\phi_j \wedge \phi_m)$.
b. $\textsc{dou}_C[\Diamond(O\phi_j \vee O\phi_m)]$ is defined only if $\Diamond O\phi_j \wedge \Diamond O\phi_m$.

(44) *$\textsc{dou}_C$ [John or Mary must teach Intro Chinese]
- FC: $\Box\phi_j \wedge \Box\phi_m$ is true relative to w, M iff j, m both teach in every world in M_w;
- RelExcl: $O\Box\phi_j$ is true relative to w, M' iff only j teaches in every world in M'_w;
- M'_w cannot be a subset of M_w unless $M'_w = \varnothing$.

This distribution explains why only $\Diamond$-questions admit FC-disjunctive answers.

A2: 'Existential' local uniqueness

The local-uniqueness inference may appear to be **existential**, especially when the *wh*-complement is modified by stressed *SINgle*:

(45) Which SINgle chapter can we assign?
$\rightsquigarrow$ *We can assign one chapter singly, aside from possibly available options of assigning more than one chapter simultaneously.*

Puzzle: Why does this local-uniqueness inference appear to be existential?
Explanation: The question nucleus is parsed with **local exhaustification**:

(46) a. Which chapter x is s.t. we can assign only x?
b. Which chapter x is s.t. we assign only x?

$$\begin{tikzcd}
 & w_1 \arrow[r, dotted, no head] & \Diamond O\phi_{c_1} \\
w \arrow[ur] \arrow[r] \arrow[dr] & w_2 \arrow[r, dotted, no head] & \Diamond O\phi_{c_2} \\
 & w_3 &
\end{tikzcd}$$

- (46a) satisfies RelExh iff (46b) has a unique true answer in every accessible world where there is a chapter x s.t. we only assign x.
- This condition is fairly weak: it only considers the accessible worlds where uniqueness is satisfied. In the assumed scenario, the set of true MS answers is $\{\Diamond O\phi_{c_1}, \Diamond O\phi_{c_2}\}$. Since none of the locally exhaustified true answers (e.g., $\Diamond O_C\phi_{c_2}$) can be verified by $\{w_3\}$, the violation of uniqueness in w_3 doesn't affect RelExh.

A3: Local uniqueness and disjunctive MA

Singular *can*-questions admit disjunctive MA interpretations. Does the RelExh-based analysis of local uniqueness apply here?

(47) Which chapter can we assign?
$\rightsquigarrow$ *We can assign exactly one chapter, but not more.* (Universal local uniqueness)
a. Chapter 1 or chapter 2.
b. #Chapter 1 and chapter 2.

Again, assume the same local-uniqueness-violating scenario:

(48) Let $M_w = \{w_1, w_2, w_3\}$, assign $= \begin{bmatrix} w_1 \to \{c_1\} \\ w_2 \to \{c_2\} \\ w_3 \to \{c_1, c_2\} \\ \ldots \end{bmatrix}$; then:
$Q^M_w = \{\textsc{dou}_C \Diamond \phi_{c_1}, \textsc{dou}_C \Diamond \phi_{c_2}, \textsc{dou}_C [\Diamond(\phi_{c_1} \vee \phi_{c_2})]\}$

RelExh alone cannot explain the observed universal local-uniqueness effect:

- The true answer $\textsc{dou}_C \Diamond \phi_{c_1}$ can be verified by $\{w_3\}$; thus, RelExh requires that the question has an ETA when interpreted relative to a modal base M' s.t. $M'_w = \{w_3\}$.
- Despite of the violation of local uniqueness, this requirement is satisfied: given any M' s.t. $M'_w = \{w_3\}$, the FC-disjunctive answer $\textsc{dou}_C [\Diamond(\phi_{c_1} \vee \phi_{c_2})]$ is an ETA in w.

A3: Local uniqueness and disjunctive MA (cont.)

Puzzle: How else then does my account avoid under-generating uniqueness in singular *can*-questions with a disjunctive MA interpretation?

Reply: Before RelExh applies, the FC-disjunctive answers have been ruled out due to violations of **Relativized Exclusivity**.

- If the prejacent sentence is parsed without local exhaustification, RelExcl yields the condition that the two distinct disjuncts cannot be simultaneously true.

(49) $\textsc{dou}_C$ [John or Mary can teach Intro Chinese]
$\textsc{dou}_C[\Diamond(\phi_j \vee \phi_m)]$ is defined only if $\Diamond O\phi_j \wedge \Diamond O\phi_m \wedge \neg\Diamond(\phi_j \wedge \phi_m)$.

- For the same reason, for the singular *can*-question (47), if $\Diamond(\phi_{c_1} \wedge \phi_{c_2})$ is true, the disjunctive answer $\textsc{dou}_C[\Diamond(\phi_{c_1} \vee \phi_{c_2})]$ is undefined.
- Once the disjunctive answers that violate RelExcl are removed from the answer space, RelExh predicts a universal local-uniqueness effect, in the same way as for MS interpretations.
- This analysis also applies to 'existential' local uniqueness.

A4. A variable-free treatment of modal bases

Problem: Kratzerian theory of modality treats the modal base as a free variable. It is unclear how the interpretation of a modalized sentence can make reference to the modal base.

(50) a. $[\![can_M\ \phi]\!]^g = \lambda w.\exists w' \in g(M)(w)[[\![\phi]\!]^g(w) = 1]$
b. $[\![should_M\ \phi]\!]^g = \lambda w.\forall w' \in g(M)(w)[[\![\phi]\!]^g(w) = 1]$

Assumption: Modalized sentences denote functions from modal bases to propositions.

(51) a. $[\![can\ \phi]\!] = \lambda M_{\langle s,st\rangle}\lambda w.\exists w' \in M_w[[\![\phi]\!](w) = 1]$
b. $[\![should\ \phi]\!] = \lambda M_{\langle s,st\rangle}\lambda w.\forall w' \in M_w[[\![\phi]\!](w) = 1]$

In analogy to the 'Geach rule' in variable-free semantics, I assume the a type-shifting operation $\mathscr{M}$ which allows a sentential operator to apply to a modalized sentence and passes up the abstraction of the modal base.

(52) For any sentential expression F of type $\langle st, \sigma\rangle$, $\mathscr{M}(F)$ is an expression of type $\langle\langle sst, st\rangle, \langle sst, \sigma\rangle\rangle$ such that $[\![\mathscr{M}(F)]\!] = \lambda\alpha_{\langle sst,st\rangle}\lambda M_{\langle s,st\rangle}.[\![F]\!](\alpha(M))$.

E.g., $\mathscr{M}$ shifts the assertion of DOU as follows:

(53) $[\![\textsc{dou}_C]\!] = \lambda p_{\langle s,t\rangle}\lambda w.p(w) = 1 \wedge \forall q \in \textsc{AntiExcl}(p, C)[O^{\text{IE}}_C(q)(w) = 0]$

(54) $[\![\mathscr{M}(\textsc{dou}_C)]\!]$
$= \lambda\theta_{\langle sst,st\rangle}\lambda M_{\langle s,st\rangle}\lambda w.\theta(M)(w) = 1 \wedge \forall q \in \textsc{AntiExcl}(\theta(M), C^M)[O^{\text{IE}}_{C^M}(q)(w) = 0]$,
where for any $M_{\langle s,st\rangle}$, $C^M := \{\rho(M) \mid \rho_{\langle sst,st\rangle} \in C\}$

A4. A variable-free treatment of modal bases (cont.)

A modalized question can now be defined as a Hamblin set, each member of which is a function from a modal base to a propositional answer.

RelExh and RelExcl can now be defined as follows:

(55) *Relativized Exhaustivity*
For any modal base M, a question Q is defined in w only if
$\forall M'_{\langle s,st\rangle}[|M'_w| = 1 \wedge M'_w \subseteq M_w \wedge \exists \theta[w \in \theta(M) \in Q^M \wedge w \in \theta(M') \in Q^{M'}] \rightarrow \mathbf{DEP}(w, M', Q)]$

a. $Q^M := \{\rho(M) \mid \rho_{\langle sst,st\rangle} \in Q\}$, where Q is a Hamblin set

b. $\mathbf{DEP}(w, M', Q) := \exists \theta[w \in \theta(M') \in Q^M \wedge \forall \delta[w \in \delta(M') \in Q^{M'} \rightarrow \theta(M') \subseteq \delta(M')]]$

(56) *Relativized Exclusivity*
For any $\theta_{\langle sst,st\rangle}$ and $M_{\langle s,st\rangle}$, $[\![\mathscr{M}(\textsc{dou}_C)]\!](\theta)(M)(w)$ is defined only if
$\forall \delta_{\langle sst,st\rangle}[\delta(M) \in \textsc{AntiExcl}(\theta(M), C^M) \wedge \delta(M) \subset \theta(M)$
$\rightarrow \forall M'[M'_w$ is a minimal subset of M_w s.t. $\delta(M')(w) = 1 \rightarrow O^{\text{IE}}_{C^{M'}}(\delta(M'))(w) = 1]]$.