Higher-order readings of *wh*-questions and the disjunction–conjunction asymmetry

Yimei Xiang
yimei.xiang@rutgers.edu
Rutgers University

MIT Linguistics Colloquium • September 25, 2020
A \textit{wh}-question calls for an answer that names an \textbf{entity} in \([\textit{wh}\text{-complement}]\) or a \textbf{generalized quantifier} (GQ) over a subset of such entities.

(1) Which student came?
   a. Andy. \(\Rightarrow\) Andy is a student.
   b. Andy or Billy. \(\Rightarrow\) Andy and Billy are students.
A *wh*-question calls for an answer that names an entity in $[[\text{wh-complement}]]$ or a **generalized quantifier** (GQ) over a subset of such entities.

(1) Which student came?
   a. Andy. $\implies$ Andy is a student.
   b. Andy or Billy. $\implies$ Andy and Billy are students.

*WhPs* are functions (e.g., $\exists$-quantifiers or $\lambda$-abstractors) over **first-order** predicates. The domain for quantification or abstraction is $[[\text{wh-complement}]]$. 

```
CP
  DP
    D  NP
      |  RESTRICTOR
      \_  student
  SCOPE
    i
    C'
  IP
    NUCLEUS
    x_i came
```
In categorial approaches ...

A wh-question denotes a function defined for entities in \(J\) wh-complement \(K\).

(2) Which student came?

\[
\lambda P:\text{stdt}(x). P(x)\]

\[
\lambda x:\text{e}. \text{came}(x)\]

\[
\lambda x:\text{stdt}(x). \text{came}(x)\]

Questions and their short answers stand in a function–argument relation:

a. Combining with an entity

\[J\ Andy K(J\ wh-Q K) = [\lambda x:\text{e}: \text{stdt}(x). \text{came}(x)](a) = \text{stdt}(a) \land \text{came}(a)\]

b. Combining with a GQ

\[J\ Andy or Billy K(J\ wh-Q K) = [a \upunion b \upunion] (\lambda x:\text{e}: \text{stdt}(x). \text{came}(x)) = \text{stdt}(a) \land \text{stdt}(b) \land \text{came}(a) \lor \text{came}(b)\]

[Q-function: the function-like question denotation; Q-domain: the domain of a Q-function.]
In categorial approaches ...

A *wh*-question denotes a function defined for entities in [*wh*-complement].
In categorial approaches ...

A *wh*-question denotes a function defined for entities in $[[wh\text{-complement}]]$.

(2) Which student came?

\[
\begin{align*}
\text{which student} & \quad \text{came} \\
[\lambda P_{et} : stdt(x).P(x)] & \quad [\lambda x.e.came(x)] \\
\lambda x_e : stdt(x).came(x)
\end{align*}
\]
A *wh*-question denotes a function defined for entities in *[[wh-complement]]*.

(2) Which student came?

\[
\text{which student came}
\]

\[
\lambda x : stdt(x).\text{came}(x)
\]

Questions and their short answers stand in a function–argument relation:

a. Combining with an entity

\[
[[\text{wh-Q}}](\text{[[Andy]]}) = \lambda x : stdt(x).\text{came}(x)(a) = stdt(a).\text{came}(a)
\]
In categorial approaches ...

A *wh*-question denotes a function defined for entities in \[[wh\text{-complement}]\].

(2) Which student came?

\[
\text{\underline{which student}} \quad \text{\underline{came}}
\]

\[
[\lambda P_{et} : \text{stdt}(x).P(x)] \quad [\lambda x.\text{came}(x)]
\]

\[
\lambda x. \text{stdt}(x).\text{came}(x)
\]

Questions and their short answers stand in a function–argument relation:

a. Combining with an entity

\[
[[\text{wh-Q}}[[\text{Andy}}] = [\lambda x. \text{stdt}(x).\text{came}(x)](a) = \text{stdt}(a).\text{came}(a)
\]

b. Combining with a GQ

\[
[[\text{Andy or Billy}}[[\text{wh-Q}}] = [a \uparrow \cup b \uparrow](\lambda x. \text{stdt}(x).\text{came}(x)) = \text{stdt}(a) \land \text{stdt}(b).\text{came}(a) \lor \text{came}(b)
\]
A *wh*-question denotes a function defined for entities in \(\llbracket \text{wh-complement} \rrbracket\).

(2) Which student came?

\[
\begin{align*}
\text{which student} & \quad \text{came} \\
[\lambda P_{et} : \text{stdt}(x).P(x)] & \quad [\lambda x_e.\text{came}(x)] \\
\lambda x_e : \text{stdt}(x).\text{came}(x)
\end{align*}
\]

Questions and their short answers stand in a function–argument relation:

a. Combining with an entity

\[
\llbracket \text{wh-Q} \rrbracket (\llbracket \text{Andy} \rrbracket) = [\lambda x_e : \text{stdt}(x).\text{came}(x)](a) = \text{stdt}(a).\text{came}(a)
\]

b. Combining with a GQ

\[
\llbracket \text{Andy or Billy} \rrbracket (\llbracket \text{wh-Q} \rrbracket) = [a \uparrow \cup b \uparrow] (\lambda x_e : \text{stdt}(x).\text{came}(x)) = \text{stdt}(a) \land \text{stdt}(b).\text{came}(a) \lor \text{came}(b)
\]

[Q-function: the function-like question denotation; Q-domain: the domain of a Q-function.]

In short:  

\[\text{Q-domain} \subseteq \llbracket \text{wh-complement} \rrbracket\]
The discussion so far has been focused on first-order readings. However, ...

In some cases, a *wh*-question must be interpreted with a higher-order (HO-)reading, in which this question calls for an answer that names a GQ.
The discussion so far has been focused on **first-order readings**. However, ...

In some cases, a *wh*-question must be interpreted with a **higher-order (HO-)reading**, in which this question calls for an answer that names a GQ.

**Goal:** To investigate the distribution and derivation of these HO-readings.
The discussion so far has been focused on first-order readings. However, ...

In some cases, a *wh*-question must be interpreted with a higher-order (HO-)reading, in which this question calls for an answer that names a GQ.

**Goal: To investigate the distribution and derivation of these HO-readings.**

- Evidence for HO-readings (Part A)
- Derivations and distributional constraints of HO-readings (esp. the ‘disjunction–conjunction asymmetry’) (Part B & C)
- Constraints on HO-answers (Part D)
Coordinations

- Disjunctions over set-denoting expressions are **unions**. Entities must be Montague-lifted before being disjoined.

\[(3) \ a. \ \llbracket Andy \ or \ Billy \rrbracket = a^{\uparrow} \cup b^{\uparrow} \]
\[b. \ \text{For any } x \text{ of type } \tau: \ x^{\uparrow} = \lambda m_{\langle \tau, t \rangle}. m(x)\]
Coordinations

- Disjunctions over set-denoting expressions are **unions**. Entities must be Montague-lifted before being disjoined.

\[(3)\]

a. \([Andy or Billy] = a^{\uparrow} \cup b^{\uparrow}\)
b. For any \(x\) of type \(\tau\): \(x^{\uparrow} = \lambda m_{(\tau,t)}.m(x)\)

- Conjunctions are ambiguous between **intersections** and **sums**.

\[(4)\]

a. \([Andy and Billy] = a^{\uparrow} \cap b^{\uparrow}\)
b. \([Andy and Billy] = a \oplus b\)
Coordinations

- Disjunctions over set-denoting expressions are **unions**. Entities must be Montague-lifted before being disjoined.

  (3)  a. \([Andy or Billy] = a⇑ ∪ b⇑\)
  b. For any \(x\) of type \(τ\): \(x⇑ = λm⟨τ,t⟩.m(x)\)

- Conjunctions are ambiguous between **intersections** and **sums**.

  (4)  a. \([Andy and Billy] = a⇑ ∩ b⇑\)
  b. \([Andy and Billy] = a ⊕ b\)

Live-on and range-over

(5) For any \(π\) of type \(⟨et, t⟩\) and any set of entities \(A\):
  a. \(π\) lives on \(A\) iff \(∀B[π(B) ⇔ π(A ∩ B)]\). (Barwise and Cooper 1981)
  b. \(π\) ranges over \(A\) iff \(A\) is the smallest live-on set (smlo) of \(π\). (Szabolcsi 1997)
Basics

Coordinations

- Disjunctions over set-denoting expressions are **unions**. Entities must be Montague-lifted before being disjoined.

  (3)  
  a. $\llbracket \text{Andy or Billy} \rrbracket = a^{\uparrow} \cup b^{\uparrow}$
  b. For any $x$ of type $\tau$: $x^{\uparrow} = \lambda m_{\tau,t}.m(x)$

- Conjunctions are ambiguous between **intersections** and **sums**.

  (4)  
  a. $\llbracket \text{Andy and Billy} \rrbracket = a^{\uparrow} \cap b^{\uparrow}$
  b. $\llbracket \text{Andy and Billy} \rrbracket = a \oplus b$

Live-on and range-over

(5) For any $\pi$ of type $\langle et, t \rangle$ and any set of entities $A$:
   a. $\pi$ lives on $A$ iff $\forall B[\pi(B) \leftrightarrow \pi(A \cap B)]$. (Barwise and Cooper 1981)
   b. $\pi$ ranges over $A$ iff $A$ is the smallest live-on set (SMLO) of $\pi$. (Szabolcsi 1997)

\[
\begin{array}{ccc}
\pi & \text{SMLO(}\pi\text{)} \\
\hline
a^{\uparrow} \cup b^{\uparrow}, a^{\uparrow} \cap b^{\uparrow} & \{a, b\}
\end{array}
\]
**Basics**

**Coordinations**

- Disjunctions over set-denoting expressions are **unions**. Entities must be Montague-lifted before being disjoined.

  (3)  
  a. \([Andy or Billy] = a \uparrow \cup b \uparrow\)  
  b. For any \(x\) of type \(\tau\): \(x \uparrow = \lambda m(\tau.t).m(x)\)

- Conjunctions are ambiguous between **intersections** and **sums**.

  (4)  
  a. \([Andy and Billy] = a \uparrow \cap b \uparrow\)  
  b. \([Andy and Billy] = a \oplus b\)

**Live-on and range-over**

(5) For any \(\pi\) of type \(\langle et, t \rangle\) and any set of entities \(A\):

  a. \(\pi\) lives on \(A\) iff \(\forall B[\pi(B) \leftrightarrow \pi(A \cap B)]\).  
     (Barwise and Cooper 1981)
  b. \(\pi\) ranges over \(A\) iff \(A\) is the smallest live-on set (smlo) of \(\pi\).  
     (Szabolcsi 1997)

<table>
<thead>
<tr>
<th>(\pi)</th>
<th>SMLO((\pi))</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a \uparrow \cup b \uparrow, a \uparrow \cap b \uparrow)</td>
<td>({a, b})</td>
<td></td>
</tr>
<tr>
<td>some/every/no book</td>
<td>book</td>
<td></td>
</tr>
<tr>
<td>some/most books</td>
<td>books</td>
<td></td>
</tr>
</tbody>
</table>
Part A. Evidence for higher-order readings

Predictions for questions with a first-order reading

If an answer names a GQ...

this GQ must be interpreted with wide scope relative to the Q-nucleus.

the proposition denoted by this answer is not in the question answer space; it is derived as a Boolean combination of the propositions in the answer space.

Both predictions have counterexamples...

Non-reducibility in questions with necessity modals: evidence for Boolean disjunctions and $\exists$-GQs

Absence of uniqueness in questions with collective predicates: evidence for Boolean conjunctions and $\forall$-GQs
Part A. Evidence for higher-order readings

Predictions for questions with a first-order reading

If an answer names a GQ ...

1. This GQ must be interpreted with **wide scope** relative to the Q-nucleus.

2. The proposition denoted by this answer is **not** in the question answer space; it is derived as a Boolean combination of the propositions in the answer space.
Part A. Evidence for higher-order readings

Predictions for questions with a first-order reading

If an answer names a GQ ...

1. this GQ must be interpreted with wide scope relative to the Q-nucleus.
2. the proposition denoted by this answer is not in the question answer space; it is derived as a Boolean combination of the propositions in the answer space.

Both predictions have counterexamples ...

1. **Non-reducibility** in questions with necessity modals: evidence for Boolean disjunctions and $\exists$-GQs
2. **Absence of uniqueness** in questions with collective predicates: evidence for Boolean conjunctions and $\forall$-GQs
Spector (2007, 2008): Disjunctions and $\exists$-GQs can completely address $\Box$-questions.

(6) Which books does John have to read?
   a. The French novels or the Russian novels.
      ‘F or R ... I don’t know which exactly.’
      ‘F or R ... the choice is up to him.’
      (Partial: $or \gg \Box$)
      (Complete: $\Box \gg or$)

(6) Which books does John have to read?
   a. The French novels or the Russian novels.
      ‘F or R ... I don’t know which exactly.’  (Partial: or ≫ □)
      ‘F or R ... the choice is up to him.’  (Complete: □ ≫ or)
   b. At least two books by Balzac.
      ‘... I don’t know what they are.’  (Partial: ∃ ≫ □)
      ‘... any two (or more).’  (Complete: □ ≫ ∃)

(6) Which books does John have to read?
   a. The French novels or the Russian novels.
      ‘F or R ... I don’t know which exactly.’ (Partial: or ⊳ □)
      ‘F or R ... the choice is up to him.’ (Complete: □ ⊳ or)
   b. At least two books by Balzac.
      ‘... I don’t know what they are.’ (Partial: ∃ ⊳ □)
      ‘... any two (or more).’ (Complete: □ ⊳ ∃)

The narrow-scope interpretation requires the question to have a HO-reading:
The whP ranges over HO-meanings and binds a HO-trace across the modal.

(7) ‘Which GQ π over books is such that John has to read π?’
    [ which-books λπ(⟨et,t⟩ [ have-to [ π λxe [ John read x ]]]) ]

(6) Which books does John have to read?
   a. The French novels or the Russian novels.
      ‘F or R ... I don’t know which exactly.’  (Partial: or ⊳ □)
      ‘F or R ... the choice is up to him.’  (Complete: □ ⊳ or)
   b. At least two books by Balzac.
      ‘... I don’t know what they are.’  (Partial: ∃ ⊳ □)
      ‘... any two (or more).’  (Complete: □ ⊳ ∃)

The narrow-scope interpretation requires the question to have a HO-reading:
The whP ranges over HO-meanings and binds a HO-trace across the modal.

(7) ‘Which GQ π over books is such that John has to read π?’
    [ which-books λπ_{(et,t)} [ have-to [ π λx e [ John read x ]]]]
    [[wh-Q]([[F or R]]) = [λπ_{(et,t)}: smlo(π) ⊆ books_@.□[λw.π(λx.read_w(j,x))]](f⇑ ∪ r⇑))
Spector (2007, 2008): Disjunctions and \( \exists \)-GQs can completely address \( \square \)-questions.

(6) Which books does John \textbf{have to} read?

a. The French novels \textbf{or} the Russian novels.

‘F or R ... I don’t know which exactly.’ \hspace{1cm} (Partial: \textbf{or} \( \gg \) \( \square \))

‘F or R ... the choice is up to him.’ \hspace{1cm} (Complete: \( \square \gg \) \textbf{or})

b. \textbf{At least two} books by Balzac.

‘... I don’t know what they are.’ \hspace{1cm} (Partial: \( \exists \gg \square \))

‘... any two (or more).’ \hspace{1cm} (Complete: \( \square \gg \exists \))

The \textbf{narrow-scope} interpretation requires the question to have a \textbf{HO-reading}:
The \textit{wh}P ranges over HO-meanings and binds a HO-trace across the modal.

(7) ‘Which GQ \( \pi \) over books is such that John has to read \( \pi \)?’

\[
\left[ \text{which-books } \lambda \pi_{\langle et, t \rangle} \right] \left[ \text{have-to } \lambda x_e \lambda \pi \left( \lambda x \ . \ \text{read}_w(j,x) \right) \right] = \left\{ f, r \right\} \subseteq \text{books}_@ \ . \ \square \left( \lambda w \ . \ \text{read}_w(j,f) \lor \text{read}_w(j,r) \right)
\]
□-questions provide evidence for Boolean disjunctions because their answer space is not closed under disjunction.

What did John read?

What does John have to read?
-questions provide evidence for Boolean disjunctions because their answer space is not closed under disjunction.

Or, the Q-function of the □-question is \textbf{not reducible} relative to disjunctions.
Diagnostic for disjunctions: Non-reducibility

□-questions provide evidence for Boolean disjunctions because their answer space is not closed under disjunction.

\[ f(a) \land f(b) \]

\[ f(a) \lor f(b) \]

What did John read?

Or, the Q-function of the □-question is **not reducible** relative to disjunctions.

For any \( \pi_{(t,t)} \), a function \( \theta \) is **reducible** relative to \( \pi \) iff \( \theta \cdot \pi \equiv \pi(\lambda x.\theta \cdot x) \).

[\( \cdot \): the combinatory operation between \( \theta \) and a GQ]
Diagnostic for disjunctions: Non-reducibility

□-questions provide evidence for Boolean disjunctions because their answer space is not closed under disjunction.

Or, the Q-function of the □-question is not reducible relative to disjunctions.

For any $\pi_{(\tau,t)}$, a function $\theta$ is reducible relative to $\pi$ iff $\theta \bullet \pi \Leftrightarrow \pi(\lambda x. \theta \bullet x)$.  

[\bullet: the combinatory operation between $\theta$ and a GQ]

(8) a. $[\lambda \pi. J \ has \ to \ read \ \pi](a \uparrow \cup b \uparrow) \nless J \ has \ to \ read \ a \lor J \ has \ to \ read \ b$

b. $[\lambda \pi. J \ has \ to \ read \ \pi](a \uparrow \cap b \uparrow) \nless J \ has \ to \ read \ a \land J \ has \ to \ read \ b$
Quantized phrasal predicates are **stubbornly collective**: they admit a collective reading but not a covered / (non-atomic) distributive reading. (E.g. *formed a team, co-authored two papers*; in contrast to *lifted the piano*).
Quantized phrasal predicates are **stubbornly collective**: they admit a collective reading but not a covered/ (non-atomic) distributive reading. (E.g. *formed a team*, *co-authored two papers*; in contrast to *lifted the piano*).

The children formed two teams in total: $a + b$ formed one team, and $c + d$ formed the other.
Quantized phrasal predicates are **stubbornly collective**: they admit a collective reading but not a covered/ (non-atomic) distributive reading. (E.g. *formed a team, co-authored two papers*; in contrast to *lifted the piano*).

The children formed two teams in total: \(a + b\) formed one team, and \(c + d\) formed the other.

(9)  
\(\begin{align*}
\text{a. } & \text{ The children formed teams.} \\
\text{b. } & \text{ #The children formed } a/\text{one team.}
\end{align*}\)
Quantized phrasal predicates are **stubbornly collective**: they admit a collective reading but not a covered/ (non-atomic) distributive reading. (E.g. *formed a team*, *co-authored two papers*; in contrast to *lifted the piano*).

The children formed two teams in total: \(a + b\) formed one team, and \(c + d\) formed the other.

(9)  
   a. The children formed teams.  
   b. #The children formed a/one team.

A contrast in embeddings:

(10) a. #John knows [that the children formed a team].
Quantized phrasal predicates are **stubbornly collective**: they admit a collective reading but not a covered / (non-atomic) distributive reading. (E.g. *formed a team*, *co-authored two papers*; in contrast to *lifted the piano*).

The children formed two teams in total: $a + b$ formed one team, and $c + d$ formed the other.

(9) a. The children formed **teams**.
   b. #The children formed **a/one team**.

A contrast in embeddings:

(10) a. #John knows [that **the children** formed a team].
   b. John knows [**which children** formed a team].
   c. $\rightsquigarrow$ John knows that $a + b$ formed a team and $c + d$ formed a team.
Quantized phrasal predicates are **stubbornly collective**: they admit a collective reading but not a covered / (non-atomic) distributive reading. (E.g. formed a team, co-authored two papers; in contrast to lifted the piano).

The children formed two teams in total: \( a + b \) formed one team, and \( c + d \) formed the other.

(9)  
a. The children formed teams.  
b. #The children formed a/one team.

A contrast in embeddings:

(10)  
a. #John knows [that the children formed a team].  
b. John knows [which children formed a team].  
c. \( \rightarrow \) John knows that \( a + b \) formed a team and \( c + d \) formed a team.

**Puzzle:** In (10c), where does the conjunctive closure come from?
Quantized phrasal predicates are **stubbornly collective**: they admit a collective reading but not a covered / (non-atomic) distributive reading. (E.g. formed a team, co-authored two papers; in contrast to lifted the piano).

The children formed two teams in total: \( a + b \) formed one team, and \( c + d \) formed the other.

(9)  
   a. The children formed teams.  
   b. #The children formed a/one team.

A contrast in embeddings:

(10)  
   a. #John knows [that the children formed a team].  
   b. John knows [which children formed a team].  
   c. \( \leadsto \) John knows that \( a + b \) formed a team and \( c + d \) formed a team.

**Puzzle:** In (10c), where does the conjunctive closure come from?  
Clearly, it isn’t from the predicate or anywhere within the question nucleus.
Xiang’s (2016) reply: This conjunctive closure is supplied by the \textit{whP}.
Xiang’s (2016) reply: This conjunctive closure is supplied by the $whP$. It binds a HO-trace and ranges over HO-meanings

(11) Which children formed a team?
‘Which GQ $\pi$ over children is such that $\pi$ formed a team?’

a. $[\text{cp which-children } \lambda \pi_{\langle et, t \rangle} [\text{ip } \pi \lambda x_{\text{e}} [\text{vp } x \text{ formed a team }]]$

b. $[wh-Q] = \lambda \pi_{\langle et, t \rangle} : \text{SMLO}(\pi) \subseteq children_{@}.\lambda w[\pi(\lambda x. \text{fm-a-tm}_{\text{w}}(x))]$
Xiang’s (2016) reply: This conjunctive closure is supplied by the \textit{whP}. It binds a HO-trace and ranges over HO-meanings including \textit{Boolean conjunctions}, e.g., $(a \oplus b)^\uparrow \cap (c \oplus d)^\uparrow$.

(11) Which children formed a team?
‘Which GQ $\pi$ over children is such that $\pi$ formed a team?’

\begin{enumerate}[	extbf{a.}]
\item [[\textit{cp} which-children $\lambda \pi_{(et,t)}$ $[\textit{ip} \ \pi \ \lambda x_e \ [\textit{vp} \ x \ formed \ a \ team ]]]$
\item \textit{[wh-Q]} = $\lambda \pi_{(et,t)} : \text{SMLO}(\pi) \subseteq \text{children}_\wedge \cdot \lambda w [\pi (\lambda x. \text{fm-a-tm}_w (x))]$
\item \textit{[wh-Q]} ($(a \oplus b)^\uparrow \cap (c \oplus d)^\uparrow$) = $\{a \oplus b, c \oplus d\} \subseteq \text{children}_\wedge \cdot \lambda w [\text{fm-a-tm}_w (a \oplus b) \land \text{fm-a-tm}_w (c \oplus d)]$
\end{enumerate}
Xiang’s (2016) reply: This conjunctive closure is supplied by the \textit{whP}. It binds a HO-trace and ranges over HO-meanings including \textit{Boolean conjunctions}, e.g., $(a \oplus b)^\uparrow \cap (c \oplus d)^\uparrow$.

(11) Which children formed a team?
‘Which GQ $\pi$ over children is such that $\pi$ formed a team?’

a. $[[cP \text{ which-children } \lambda \pi_{\langle et, t \rangle} [\text{ip } \pi \lambda x e [\text{vp } x \text{ formed a team }]]]]$

b. $[[wh-Q]] = \lambda \pi_{\langle et, t \rangle} : \text{SMLO}(\pi) \subseteq \text{children@}.\lambda w[\pi(\lambda x. \text{fm-a-tm}_w(x))]$

c. $[[wh-Q]](\langle a \oplus b \rangle^\uparrow \cap (c \oplus d)^\uparrow) = \{a \oplus b, c \oplus d\} \subseteq \text{children@}.\lambda w[\text{fm-a-tm}_w(a \oplus b) \land \text{fm-a-tm}_w(c \oplus d)]$

A plausible alternative (Fox 2018, 2020):
The \textit{whP} can range over HO-pluralities, e.g., $\{\{a, b\}, \{c, d\}\}$. 
Another attempt: The conjunctive closure is from an operation outside the Q-root, such as the answerhood-operator.

\[(12) \quad \text{Ans}_{\text{Heim}}(w)(Q) = \bigcap \{p \mid w \in p \in Q\} \quad \text{(Heim 1994)}\]
Another attempt: The conjunctive closure is from an operation outside the Q-root, such as the answerhood-operator.

\[
\text{ANS}_{\text{Heim}}(w)(Q) = \bigcap \{p \mid w \in p \in Q\}
\]

(12) \hspace{1cm} (\text{Heim 1994})

Infeasible. The following contrast w.r.t. uniqueness argues that the conjunctive closure is related to the \textit{whP}:

(13) (Among the children, \(a + b\) formed a team, \(c + d\) formed another team.)
   a. John knows [which \textit{children} formed a team].
   b. #John knows [which \textit{two children} formed a team].
Another attempt: The conjunctive closure is from an operation outside the Q-root, such as the answerhood-operator.

\[(12) \quad \text{Ans}_{\text{Heim}}(w)(Q) = \bigcap \{ p \mid w \in p \in Q \} \quad \text{(Heim 1994)}\]

Infeasible. The following contrast w.r.t. \textit{uniqueness} argues that the conjunctive closure is related to the \textit{whP}:

\[(13) \quad \text{(Among the children, } a + b \text{ formed a team, } c + d \text{ formed another team.)}\]

\ a. John knows \text{ [which children formed a team].}\n\ b. #John knows \text{ [which two children formed a team].}\n\ \rightarrow \text{Only two of the children formed any team.}\]
Another attempt: The conjunctive closure is from an operation outside the Q-root, such as the answerhood-operator.

\[(12) \quad \text{ANS}_{\text{Heim}}(w)(Q) = \bigcap \{p \mid w \in p \in Q\} \quad \text{(Heim 1994)}\]

Infeasible. The following contrast w.r.t. uniqueness argues that the conjunctive closure is related to the \text{whP}:

\[(13) \quad \text{(Among the children, } a + b \text{ formed a team, } c + d \text{ formed another team.)}\]

a. John knows [which \textit{children} formed a team].

b. #John knows [which \textit{two children} formed a team].

\[\Rightarrow \text{Only two of the children formed any team.}\]

\[\Rightarrow \text{The \textit{numeral-modifier} in the } \text{whP} \text{ makes HO-conjunctive answers unavailable and yields uniqueness.}\]
Diagnostic for conjunctions: A note on uniqueness

Dayal (1996): A question is defined only if it has a strongest true answer.

\[ f(a \oplus b) \land f(c \oplus d) \]

... 

Which children formed a team?

The contrast w.r.t. uniqueness is expected if ...

- Simplex plural \(wh\)Ps can quantify over HO-conjunctions, (bare \(wh\)Ps likewise)
Dayal (1996): A question is defined only if it has a strongest true answer.

\[ f(a \oplus b) \land f(c \oplus d) \]

Which two children formed a team?

The contrast w.r.t. uniqueness is expected if ...

- Simplex plural whPs can quantify over HO-conjunctions, (bare whPs likewise)
- while numeral-modified whPs cannot. (singular whPs likewise)
Dayal (1996): A question is defined only if it has a strongest true answer.

\[ f(a \oplus b) \land f(c \oplus d) \]

Which two children formed a team?

The contrast w.r.t. uniqueness is expected if ...
- Simplex plural whPs can quantify over HO-conjunctions, (bare whPs likewise)
- while numeral-modified whPs cannot. (singular whPs likewise)

Other explanations of uniqueness
- Presuppositional which (Rullmann & Beck 1998; Uegaki 2020; Hirsh & Schwarz 2020)
- Question–Partition Matching (Fox 2018, 2020; Kobayashi & Rouillard 2020)
- Relativized Exhaustivity (Xiang 2020c)
• HO-readings arise if the \( wh \)P ranges over HO-meanings and binds a HO-trace.

```
   CP
   /\        /
   /  \    /  \    
   DP  \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
       \  \  \  \    
```

- Boolean disjunctions and \( \exists \)-GQs (by non-reducibility w.r.t. GQ-naming answers in \( \Box \)-questions)
- Boolean conjunctions and \( \forall \)-GQs (by absence of uniqueness in questions with a stubbornly collective predicate)
- Boolean coordinations of those above
• HO-readings arise if the whP ranges over HO-meanings and binds a HO-trace.

• In a HO-reading, the Q-domain includes at least the following:

  • Boolean disjunctions and \( \exists \)-GQs
    (by non-reducibility w.r.t. GQ-naming answers in \( \Box \)-questions)
  • Boolean conjunctions and \( \forall \)-GQs
    (by absence of uniqueness in questions with a stubbornly collective predicate)
  • Boolean coordinations of those above
Part B. Distributing higher-order readings
• Uniqueness effects show that singular and numeral-modified *wh*-questions cannot have answers naming Boolean conjunctions.

(14) I know \[\text{which book}\] John read,
    ... Book A.
    #... Book A and Book B.

(15) I know \[\text{which two children}\] formed a team,
    ... the two boys.
    #... the two boys and the two girls.

Conjectures (to be possibly refined)
• Singular and numeral-modified *wh*-Ps cannot range over HO-meanings.
• The formation of a HO-domain is sensitive to the internal structure of the *wh*-complement.
• Uniqueness effects show that singular and numeral-modified *wh*-questions cannot have answers naming Boolean conjunctions.

(14) I know [which book] John read,
    ... Book A.
    #... Book A and Book B.
(15) I know [which two children] formed a team,
    ... the two boys.
    #... the two boys and the two girls.

Conjectures (to be possibly refined)

• Singular and numeral-modified *wh*Ps cannot range over HO-meanings.
Distributional constraints of HO-readings

- Uniqueness effects show that singular and numeral-modified \textit{wh}-questions cannot have answers naming Boolean conjunctions.

(14) I know \underline{which book} John read,
    ...
    Book A.
    #... Book A and Book B.

(15) I know \underline{which two children} formed a team,
    ...
    the two boys.
    #... the two boys and the two girls.

- In contrast to numeral-modifiers, PP-modifiers do not trigger uniqueness.

(16) I know \underline{which children (who are) in a group of two} formed a team,
    ...
    the two boys.
    ...
    the two boys and the two girls.

\textbf{Conjectures} (to be possibly refined)

- Singular and numeral-modified \textit{wh}Ps cannot range over HO-meanings.
Uniqueness effects show that singular and numeral-modified \(wh\)-questions cannot have answers naming Boolean conjunctions.

\[(14)\] I know [**which** book] John read,

... Book A.

#... Book A and Book B.

\[(15)\] I know [**which two children**] formed a team,

... the two boys.

#... the two boys and the two girls.

In contrast to numeral-modifiers, PP-modifiers do not trigger uniqueness.

\[(16)\] I know [**which children (who are)** in a group of two] formed a team,

... the two boys.

... the two boys and the two girls.

**Conjectures** (to be possibly refined)

- Singular and numeral-modified \(wh\)Ps cannot range over HO-meanings.
- The formation of a HO-domain is sensitive to the internal structure of the \(wh\)-complement.
Predicting the distribution

Assumptions on NP structure are from Sauerland (2003), Harbour (2014), Scontras (2014).
Predicting the distribution

Assumptions on NP structure are from Sauerland (2003), Harbour (2014), Scontras (2014).
Predicting the distribution

[Assumptions on NP structure are from Sauerland (2003), Harbour (2014), Scontras (2014).]
Predicting the distribution

[Assumptions on NP structure are from Sauerland (2003), Harbour (2014), Scontras (2014).]
Predicting the distribution

An \textbf{h-shifter} is applied to \textit{nP}, which denotes a set with a semi-lattice structure.

[Assumptions on NP structure are from Sauerland (2003), Harbour (2014), Scontras (2014).]
An \textit{h-shifter} is applied to $nP$, which denotes a set with a semi-lattice structure. It converts a set of \textit{entities} into a set of \textbf{GQs} ranging over subsets of these entities.

$$[[H]] = \lambda A_{(e,t)} \cdot \{ \pi_{(et,t)} \mid \text{SMLO}(\pi) \subseteq A \}$$

[Assumptions on NP structure are from Sauerland (2003), Harbour (2014), Scontras (2014).]
1. The $\mathbf{h}$-shifter cannot be used in **singular** nouns: applying $\mathbf{h}$ to $nP$ returns a set of GQs, which are **non-atomic** and conflict with the **atomicity** requirement of $[\text{sg}]$.

(17)  

\[
\forall \phi P \\
\phi^0 \\
\mathbf{H} \\
[\text{sg}] \\
n^0 \sqrt{\text{child}} \\
\]

\[
[[\text{sg}]] = \lambda P_{(e,t)} \lambda x_e. \text{ATOM}(x) \land P(x)
\]

[NB: A phrase that is morphologically singular (e.g., Spanish *quién* ‘who.sg’) can be number-neutral in semantics (Maldonado 2017/2020, Alonso-Ovalle & Rouillard 2019; cf. Elliott et al. 2020)]
1. The \( h \)-shifter cannot be used in **singular** nouns: applying \( h \) to \( nP \) returns a set of GQs, which are **non-atomic** and conflict with the **atomicity** requirement of [\( sg \)].

\[
\begin{align*}
\text{(17)} \quad \text{child} & \quad \text{children} \\
\end{align*}
\]

\[
\begin{array}{c}
\text{[[sg]]} = \lambda P_{(e,t)} \lambda x_e. \text{ATOM}(x) \land P(x) \\
\text{[[pl]]} = \lambda P_{(e,t)} \lambda x_e. P(x)
\end{array}
\]

[NB: A phrase that is morphologically singular (e.g., Spanish *quién* ‘who.sg’) can be number-neutral in semantics (Maldonado 2017/2020, Alonso-Ovalle & Rouillard 2019; cf. Elliott et al. 2020)]
2. The $h$-shifter cannot be used in **numeral-modified** nouns:
   The `CARD`-predicate checks the **cardinality** of the elements in the set it combines with; hence, it cannot combine with a set of GQs.

   (18) *two children*

\[
\begin{array}{c}
\phi P \\
\phi^0 \\
\hline
[PL] \\
\hline
\text{Numeral} \\
\hline
\text{Num}^0 \\
\hline
\text{two} \\
\hline
\text{CARD} \\
\hline
\mathcal{X}_H \\
\hline
n^0 \\
\hline
\sqrt{\text{child}}
\end{array}
\]

\[
\llbracket \text{CARD} \rrbracket = \lambda P \lambda N \lambda x. P(x) \land |x| = N
\]
3. **PP-modifiers** are adjoined to the entire NP/φP; hence, the h-shifter can be used within the modified NP without causing type mismatch.

(19) *children in a group of two*
• Derivation
  – Applying the $h$-shifter to the $nP$ within the $wh$-complement returns a $wh$-restrictor denoting a set of GQs.
  – The fronted $whP$ binds a HO-trace in the question nucleus.
Interim summary

**Derivation**

- Applying the $h$-shifter to the $nP$ within the $wh$-complement returns a $wh$-restrictor denoting a set of GQs.
- The fronted $whP$ binds a HO-trace in the question nucleus.

**Distribution** (to be possibly emended):
If the $wh$-complement is singular or numeral-modified, the atomicity and cardinality requirements block the application of $h$, making HO-readings unavailable.
Part C. The disjunction–conjunction asymmetry

Modalized questions with a singular or numeral-modified *wh*-phrase rejects elided conjunctive answers but admits disjunctive answers.
The puzzles: for □-questions

- Part B:
  Singular and numeral-modified whP don’t license HO-readings, because the application of h is blocked by the atomicity/cardinality requirement of [SG]/CARD.
The puzzles: for □-questions

- Part B:
  Singular and numeral-modified whP don’t license HO-readings, because the application of $h$ is blocked by the atomicity/cardinality requirement of $[\text{sg}]$/\text{CARD}.

- However, as direct answers to singular or numeral-modified □-questions, elided disjunctions are more acceptable than elided conjunctions.

(20) I know which book John has to read, ...

  #... Book A and Book B.
  ?... Book A or Book B.  (#or $\gg$ □, ?□ $\gg$ or)
The puzzles: for □-questions

• Part B:
  Singular and numeral-modified whP don’t license HO-readings, because the application of $h$ is blocked by the atomicity/cardinality requirement of $[\text{sg}]/\text{CARD}$.

• However, as direct answers to singular or numeral-modified □-questions, elided disjunctions are more acceptable than elided conjunctions.

(20) I know which book John has to read, ...
    #... Book A and Book B.
    ?... Book A or Book B.  (#or $\gg$ □, ?□ $\gg$ or)

(21) I know which two books John has to read ...
    ??... the two F books and the two R books.
    ?... the two F books or the two R books.  (#or $\gg$ □, ?□ $\gg$ or)
The puzzles: for □-questions

- Part B:
  Singular and numeral-modified *whP* don’t license HO-readings, because the application of *H* is blocked by the atomicity/cardinality requirement of [sg]/card.

- However, as direct answers to singular or numeral-modified □-questions, elided disjunctions are more acceptable than elided conjunctions.

(20) I know which *book* John has to read, ...
  
  #... Book A **and** Book B.  
  ?... Book A **or** Book B.  

(21) I know which *two books* John has to read ...
  
  #... the two F books **and** the two R books.  
  ?... the two F books **or** the two R books.  

Especially in discourse:

(22) Which *textbook* should I use for this class?  
  *Heim & Kratzer* **or** *Meaning & Grammar*. The choice is up to you.  
  
  (□ ≫ or)
The puzzles: for □-questions

• Part B:
  Singular and numeral-modified whP don’t license HO-readings, because the application of \( h \) is blocked by the atomicity/cardinality requirement of \([\text{sg}]/\text{CARD}\).

• However, as direct answers to singular or numeral-modified □-questions, elided disjunctions are more acceptable than elided conjunctions.

(20) I know which book John has to read, ...
  #... Book A and Book B.
  ?... Book A or Book B.  (#or \(\gg\) □, ?□ \(\gg\) or)

(21) I know which two books John has to read ...
  ??... the two F books and the two R books.
  ?... the two F books or the two R books.  (#or \(\gg\) □, ?□ \(\gg\) or)

Especially in discourse:

(22) Which textbook should I use for this class?
  *Heim & Kratzer or Meaning & Grammar.* The choice is up to you.  (□ \(\gg\) or)

### Puzzles

1. Why disjunctive HO-answers are acceptable despite that the wh-complement is singular or numeral-modified?
2. Why these questions admit elided disjunctions but not elided conjunctions?
The puzzles: for ♦-questions

Background:

- The mention-all (MA-)answer to a ♦-question can be expressed either as an elided conjunction or as a free-choice (FC) disjunction.

(23) What can I use (as a textbook) for this class?
   a. Heim & Kratzer and Meaning & Grammar. Conj-MA
   b. Heim & Kratzer or Meaning & Grammar. (The choice is up to you.) Disj-MA
The puzzles: for ◊-questions

Background:

• The mention-all (MA-)answer to a ◊-question can be expressed either as an elided conjunction or as a free-choice (FC) disjunction.

(23) What can I use (as a textbook) for this class?
   a. Heim&Kratzer and Meaning&Grammar. Conj-MA
   b. Heim&Kratzer or Meaning&Grammar. (The choice is up to you.) Disj-MA

• Xiang (2016, 2020c): Readings accepting conj-MA and readings accepting disj-MA are derived from different LF-s with distinct Q-nucleus.
The puzzles: for ♦-questions

Background:

- The mention-all (MA-)answer to a ♦-question can be expressed either as an elided conjunction or as a free-choice (FC) disjunction.

(23) What can I use (as a textbook) for this class?
   a. Heim&Kratzer and Meaning&Grammar. Conj-MA
   b. Heim&Kratzer or Meaning&Grammar. (The choice is up to you.) Disj-MA

- Xiang (2016, 2020c): Readings accepting conj-MA and readings accepting disj-MA are derived from different LFs with distinct Q-nucleus. The whP binds a HO-trace $\pi$, 

\[ J/d.sc/o.sc/C_K = \lambda p_\lambda w : p(w) = 1 \land \forall q \in S/u/sc/b.sc(p, C)\{O_C(q)(w) = 0\}, \text{where} \]

\[ S/u.sc/b.sc(p, C) = (C - \{p\}) - I/e.sc/x.sc/c.sc/l.sc(p, C) \] (cf. Fox's (2007) recursive exhaustification, Chierchia's (2013) pre-exhaustification exhaustifier)
The puzzles: for ♦-questions

Background:

• The mention-all (MA-)answer to a ♦-question can be expressed either as an elided conjunction or as a free-choice (FC) disjunction.

(23) What can I use (as a textbook) for this class?
   a. Heim & Kratzer and Meaning & Grammar. Conj-MA
   b. Heim & Kratzer or Meaning & Grammar. (The choice is up to you.) Disj-MA

• Xiang (2016, 2020c): Readings accepting conj-MA and readings accepting disj-MA are derived from different LFs with distinct Q-nucleus. The whP binds a HO-trace π,
  • Conj-MA is accepted when π takes wide scope,
The puzzles: for ♦-questions

Background:

- The mention-all (MA-)answer to a ♦-question can be expressed either as an elided conjunction or as a free-choice (FC) disjunction.

(23) What can I use (as a textbook) for this class?
   a. Heim & Kratzer and Meaning & Grammar. Conj-MA
   b. Heim & Kratzer or Meaning & Grammar. (The choice is up to you.) Disj-MA

- Xiang (2016, 2020c): Readings accepting conj-MA and readings accepting disj-MA are derived from different LF-s with distinct Q-nucleus. The whP binds a HO-trace π,
  - Conj-MA is accepted when π takes wide scope,
  - Disj-MA is accepted when π is associated with a FC-triggering operator dou (≈ the Mandarin particle dou, see Xiang 2020a), regardless of the scope of π.
The puzzles: for ♦-questions

Background:

• The mention-all (MA-)answer to a ♦-question can be expressed either as an elided conjunction or as a free-choice (FC) disjunction.

(23) What can I use (as a textbook) for this class?

a. Heim & Kratzer and Meaning & Grammar. Conj-MA
b. Heim & Kratzer or Meaning & Grammar. (The choice is up to you.) Disj-MA

• Xiang (2016, 2020c): Readings accepting conj-MA and readings accepting disj-MA are derived from different LFIs with distinct Q-nucleus. The whP binds a HO-trace π,

  - Conj-MA is accepted when π takes wide scope,
  - Disj-MA is accepted when π is associated with a FC-triggering operator dou (≈ the Mandarin particle dou, see Xiang 2020a), regardless of the scope of π.

\[
\begin{align*}
\text{DOU} & \left[ \Diamond (\phi \lor \psi) \right] \\
\text{DOU} & \left[ \Diamond (\phi \lor \Diamond \psi) \right] \\
\text{DOU} & \left[ \Diamond (\phi \land \Diamond \psi) \right] \\
\end{align*}
\]

\[\iff \Diamond \phi \land \Diamond \psi\]

\[\text{DOUC} = \lambda p \lambda w : p(w) = 1 \land \forall q \in \text{SUB}(p, C)[O_C(q)(w) = 0], \text{where } \text{SUB}(p, C) = (C - \{p\}) - \text{IEXCL}(p, C)\]

(cf. Fox’s (2007) recursive exhaustification, Chierchia’s (2013) pre-exhaustification exhaustifier)
a. $\pi \gg \Diamond$ (without $\text{dou}$): $\text{conj-MA}$

\[
\begin{array}{c}
\text{CP} \\
\ldots \\
\pi_{(et,t)} \\
\lambda x \\
\Diamond \text{Of}(x_e)
\end{array}
\]

\[
\begin{array}{c}
\Diamond \text{Of}(a) \\
\wedge \\
\Diamond \text{Of}(b)
\end{array}
\]

\[
\begin{array}{c}
\Diamond \text{Of}(a) \\
\vee \\
\Diamond \text{Of}(b)
\end{array}
\]
a. $\pi \gg \Box$ (without DOU): conj-MA

\[
\begin{array}{c}
\text{CP} \\
\quad ... \\
\quad \pi_{\langle et, t \rangle} \\
\quad \quad \lambda x \quad \Box \text{Of}(x_e)
\end{array}
\]

b. $\pi \gg \Box$ (with DOU): conj/disj-MA

\[
\begin{array}{c}
\text{CP} \\
\quad ... \\
\quad \pi_{\langle et, t \rangle} \\
\quad \quad \lambda x \quad \Box \text{Of}(x_e)
\end{array}
\]

\[
\begin{array}{c}
\text{DOU} \\
\quad \pi_{\langle et, t \rangle} \\
\quad \quad \lambda x \quad \Box \text{Of}(x_e)
\end{array}
\]

\[
\begin{array}{c}
\Box \text{Of}(a) \\
\quad \land \\
\quad \Box \text{Of}(b)
\end{array}
\]

\[
\begin{array}{c}
\Box \text{Of}(a) \\
\quad \lor \\
\quad \Box \text{Of}(b)
\end{array}
\]

\[
\begin{array}{c}
\text{DOU} \Box \text{Of}(a) \\
\quad \land \\
\quad \text{DOU} \Box \text{Of}(b)
\end{array}
\]

\[
\begin{array}{c}
\text{DOU} \Box \text{Of}(a) \\
\quad \lor \\
\quad \text{DOU} \Box \text{Of}(b)
\end{array}
\]
a. $\pi \gg \diamond$ (without $\text{dou}$): $\text{conj-MA}$

\[\text{CP} \quad \text{IP} \quad \pi_{\langle t, t \rangle} \quad \lambda x \quad \diamond \text{Of}(x_e)\]

b. $\pi \gg \diamond$ (with $\text{dou}$): $\text{conj/disj-MA}$

\[\text{DOU} \quad \text{CP} \quad \text{IP} \quad \pi_{\langle t, t \rangle} \quad \lambda x \quad \diamond \text{Of}(x_e)\]

c. $\diamond \gg \pi$ (with $\text{dou}$): $\text{disj-MA}$

\[\text{DOU} \quad \text{CP} \quad \text{IP} \quad \pi_{\langle t, t \rangle} \quad \lambda x \quad \text{Of}(x_e)\]
The puzzles: for ♦-questions

The puzzles:

• Hirsch & Schwarz (2020): A singular ♦-question admits a multi-choice reading if the uniqueness inference evoked by the singular whP has a narrow scope.

(24) Which letter could be missing in fo___m? (♦ ≫ ɪ)

‘Which x is such that x could be the unique letter missing in fo___m?’

The unique missing letter could be a, and the unique missing letter could be r.
The puzzles: for ♦-questions

- Hirsch & Schwarz (2020): A singular ♦-question admits a **multi-choice** reading if the uniqueness inference evoked by the singular *whP* has a **narrow scope**.

  (24) Which letter could be missing in *fo__m*? (♦ ≫ ı)
  ‘Which *x* is such that *x* could be the unique letter missing in *fo__m*?’
  The unique missing letter could be *a*, and the unique missing letter could be *r*.

- Crucially, **elided** multi-choice answers to singular ♦-questions must be disjunctions.

  (25)  
  a. Which letter could be missing in *fo__m*?
      *a* # and /or *r*.
  b. Which textbook can I use for this class?
      *H&K* # and /or *M&G*. 
The puzzles: for ♦-questions

- Hirsch & Schwarz (2020): A singular ♦-question admits a multi-choice reading if the uniqueness inference evoked by the singular whP has a narrow scope.

(24) Which letter could be missing in $fo_\_m$? (♦ ≫ 1)
‘Which $x$ is such that $x$ could be the unique letter missing in $fo_\_m$?’
The unique missing letter could be $a$, and the unique missing letter could be $r$.

- Crucially, elided multi-choice answers to singular ♦-questions must be disjunctions.

(25) a. Which letter could be missing in $fo_\_m$?
   $a$ # and/or $r$.

   b. Which textbook can I use for this class?
   $H&K$ # and/or $M&G$.

Similarly, in numeral-modified ♦-questions:

(26) [Hearing a rhotic back vowel] Which two letters could be missing in $f_\_\_m$?
$or ?? and/or ar$. 
Also for *how many*-questions:

- Gentile & Schwarz (2018): In *how many*-questions, ◊-modals can obviate violations of global uniqueness.
The puzzles: ◊-questions

Also for how many-questions:

- Gentile & Schwarz (2018): In how many-questions, ◊-modals can obviate violations of global uniqueness.

Uniqueness:

(27) How many students solved this problem together?
    # Two and three.
    (Intended: ‘Two students solved this problem together, and (another) three students solved this problem together.’)
The puzzles: $\lozenge$-questions

Also for *how many*-questions:

- Gentile & Schwarz (2018): In *how many*-questions, $\lozenge$-modals can obviate violations of global uniqueness.

Uniqueness:

(27) How many students solved this problem together?
   # Two and three.
   (Intended: ‘Two students solved this problem together, and (another) three students solved this problem together.’)

Modal obviation:

(28) How many students are allowed to solve this problem together?
    Two are OK and three are OK.
Also for *how many*-questions:

- Gentile & Schwarz (2018): In *how many*-questions, ◊-modals can obviate violations of global uniqueness.

Uniqueness:

(27) How many students solved this problem together?
    # Two and three.
    (Intended: ‘Two students solved this problem together, and (another) three students solved this problem together.’)

Modal obviation:

(28) How many students are allowed to solve this problem together?
    Two are OK and three are OK.

Similarly, the multi-choice answer cannot be expressed by an elided conjunction:

(29) How many students are allowed to solve this problem together?
    Two #and/or three.
### Puzzles

1. Why disjunctive HO-answers are acceptable despite that the *wh*-complement is singular or numeral-modified?
2. What causes the disj–conj asymmetry in the HO-readings of these questions?
The puzzles

Puzzles

1. Why disjunctive HO-answers are acceptable despite that the *wh*-complement is singular or numeral-modified?

2. What causes the disj–conj asymmetry in the HO-readings of these questions?

3. How come the uniqueness inference can be local?

Two directions

• Special-reading (A reconstruction approach, see Appendix): The ‘conj-rejecting’ reading is a special HO-reading which makes use of additional devices (e.g., syntactic reconstruction) and allows uniqueness to be interpreted local.

• Single-reading (A uniform approach): The ‘conj-admitting’ and ‘conj-rejecting’ readings are the same HO-reading. Some Boolean disjunctions are atomic/cardinal. (Uniqueness can be interpreted local for an independent reason, see Xiang 2020c.)
The puzzles

Puzzles

1. Why disjunctive HO-answers are acceptable despite that the \(wh\)-complement is singular or numeral-modified?
2. What causes the disj–conj asymmetry in the HO-readings of these questions?
3. How come the uniqueness inference can be local?

Two directions

- Special-reading (A reconstruction approach, see Appendix): The ‘conj-rejecting’ reading is a special HO-reading which makes use of additional devices (e.g., syntactic reconstruction) and allows uniqueness to be interpreted local.
The puzzles

Puzzles

1. Why disjunctive HO-answers are acceptable despite that the \textit{wh}-complement is singular or numeral-modified?

2. What causes the disj–conj asymmetry in the HO-readings of these questions?

3. How come the uniqueness inference can be local?

Two directions

- **Special-reading** (A reconstruction approach, see Appendix): The ‘conj-rejecting’ reading is a special HO-reading which makes use of additional devices (e.g., syntactic reconstruction) and allows uniqueness to be interpreted local.

- **Single-reading** (A uniform approach): The ‘conj-admitting’ and ‘conj-rejecting’ readings are the same HO-reading. Some Boolean disjunctions are atomic/cardinal. (Uniqueness can be interpreted local for an independent reason, see Xiang 2020c.)
Proposal: Just like entities, disjunctions (but not conjunctions) can be atomic/cardinal. (The direction of this proposal comes from a personal communication with Manuel Križ.)
A uniform analysis

**Proposal:** Just like entities, disjunctions (but not conjunctions) can be atomic/cardinal. (The direction of this proposal comes from a personal communication with Manuel Križ.)

<table>
<thead>
<tr>
<th>GQ</th>
<th>minimal witness set(s)</th>
<th>smallest live-on set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^\uparrow$</td>
<td>${a}$</td>
<td>${a}$</td>
</tr>
<tr>
<td>$a^\uparrow \cap b^\uparrow$</td>
<td>${a, b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>$a^\uparrow \cup b^\uparrow$</td>
<td>${a}$, ${b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>$(b \oplus c)^\uparrow$</td>
<td>${b \oplus c}$</td>
<td>${b \oplus c}$</td>
</tr>
<tr>
<td>$a^\uparrow \cup (b \oplus c)^\uparrow$</td>
<td>${a}$, ${b \oplus c}$</td>
<td>${a, b \oplus c}$</td>
</tr>
</tbody>
</table>

- For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is **atomic** iff
Proposal: Just like entities, disjunctions (but not conjunctions) can be atomic/cardinal. (The direction of this proposal comes from a personal communication with Manuel Križ.)

<table>
<thead>
<tr>
<th>GQ</th>
<th>minimal witness set(s)</th>
<th>smallest live-on set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^\uparrow$</td>
<td>${a}$</td>
<td>${a}$</td>
</tr>
<tr>
<td>$a^\uparrow \cap b^\uparrow$</td>
<td>${a, b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>$a^\uparrow \cup b^\uparrow$</td>
<td>${a}, {b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>$(b \oplus c)^\uparrow$</td>
<td>${b \oplus c}$</td>
<td>${b \oplus c}$</td>
</tr>
<tr>
<td>$a^\uparrow \cup (b \oplus c)^\uparrow$</td>
<td>${a}, {b \oplus c}$</td>
<td>${a, b \oplus c}$</td>
</tr>
</tbody>
</table>

- For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is **atomic** iff
  a. every minimal witness set of $\pi$ is a *singleton* set,
A uniform analysis

**Proposal:** Just like entities, disjunctions (but not conjunctions) can be atomic/cardinal. (The direction of this proposal comes from a personal communication with Manuel Križ.)

<table>
<thead>
<tr>
<th>GQ</th>
<th>minimal witness set(s)</th>
<th>smallest live-on set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^\uparrow$</td>
<td>${a}$</td>
<td>${a}$</td>
</tr>
<tr>
<td>$a^\uparrow \cap b^\uparrow$</td>
<td>${a, b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>$a^\uparrow \cup b^\uparrow$</td>
<td>${a}, {b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>$(b \oplus c)^\uparrow$</td>
<td>${b \oplus c}$</td>
<td>${b \oplus c}$</td>
</tr>
<tr>
<td>$a^\uparrow \cup (b \oplus c)^\uparrow$</td>
<td>${a}, {b \oplus c}$</td>
<td>${a, b \oplus c}$</td>
</tr>
</tbody>
</table>

- For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is **atomic** iff
  a. every minimal witness set of $\pi$ is a **singleton** set,
**Proposal:** Just like entities, disjunctions (but not conjunctions) can be atomic/cardinal. (The direction of this proposal comes from a personal communication with Manuel Križ.)

<table>
<thead>
<tr>
<th>GQ</th>
<th>minimal witness set(s)</th>
<th>smallest live-on set</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \uparrow )</td>
<td>{a}</td>
<td>{a}</td>
</tr>
<tr>
<td>( a \uparrow \land b \uparrow )</td>
<td>{a, b}</td>
<td>{a, b}</td>
</tr>
<tr>
<td>( a \uparrow \lor b \uparrow )</td>
<td>{a}, {b}</td>
<td>{a, b}</td>
</tr>
<tr>
<td>( (b \oplus c) \uparrow )</td>
<td>{b \oplus c}</td>
<td>{b \oplus c}</td>
</tr>
<tr>
<td>( a \uparrow \lor (b \oplus c) \uparrow )</td>
<td>{a}, {b \oplus c}</td>
<td>{a, b \oplus c}</td>
</tr>
</tbody>
</table>

- For any \( \pi \) of type \( \langle \text{et}, t \rangle \), \( \pi \) is **atomic** iff
  a. every minimal witness set of \( \pi \) is a **singleton** set,
  b. every member in the smallest live-on set of \( \pi \) is **atomic**.
**Proposal:** Just like entities, disjunctions (but not conjunctions) can be atomic/cardinal. (The direction of this proposal comes from a personal communication with Manuel Križ.)

<table>
<thead>
<tr>
<th>GQ</th>
<th>minimal witness set(s)</th>
<th>smallest live-on set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>{a}</td>
<td>{a}</td>
</tr>
<tr>
<td>$x\ a \cap b$</td>
<td>{a, b}</td>
<td>{a, b}</td>
</tr>
<tr>
<td>$a \cup b$</td>
<td>{a}, {b}</td>
<td>{a, b}</td>
</tr>
<tr>
<td>$x\ (b \oplus c)$</td>
<td>{b \oplus c}</td>
<td>{b \oplus c}</td>
</tr>
<tr>
<td>$x\ a \cup (b \oplus c)$</td>
<td>{a}, {b \oplus c}</td>
<td>{a, b \oplus c}</td>
</tr>
</tbody>
</table>

- For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is **atomic** iff
  a. every minimal witness set of $\pi$ is a **singleton** set,
  b. every member in the smallest live-on set of $\pi$ is **atomic**.
**Proposal:** Just like entities, disjunctions (but not conjunctions) can be atomic/cardinal. (The direction of this proposal comes from a personal communication with Manuel Križ.)

<table>
<thead>
<tr>
<th>GQ</th>
<th>minimal witness set(s)</th>
<th>smallest live-on set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^\uparrow$</td>
<td>${a}$</td>
<td>${a}$</td>
</tr>
<tr>
<td>$\times a^\uparrow \cap b^\uparrow$</td>
<td>${a, b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>$a^\uparrow \cup b^\uparrow$</td>
<td>${a}, {b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>$\times (b \oplus c)^\uparrow$</td>
<td>${b \oplus c}$</td>
<td>${b \oplus c}$</td>
</tr>
<tr>
<td>$\times a^\uparrow \cup (b \oplus c)^\uparrow$</td>
<td>${a}, {b \oplus c}$</td>
<td>${a, b \oplus c}$</td>
</tr>
</tbody>
</table>

- For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is **atomic** iff
  a. every minimal witness set of $\pi$ is a **singleton** set,
  b. every member in the smallest live-on set of $\pi$ is **atomic**.

- For any $\pi$ of type $\langle et, t \rangle$, $\pi$ has the **cardinality** $N$ iff
  a. every minimal witness set of $\pi$ is a singleton set,
  b. every member in the smallest live-on set of $\pi$ has the **cardinality** $N$. 

The $h$-shifter can be applied to the $nP$ of any noun. In singular/numeral-modified nouns, $[sg]/$card rules out most GQs except a few Boolean disjunctions.
The **n**-shifter can be applied to the **nP** of any noun. In singular/numeral-modified nouns, **[sg]/card** rules out most GQs except a few Boolean disjunctions.

(30) a. child

\[
\phi_P \\
\phi^0 \\
(\text{H}) \\
[\text{[sg]}] \\
n^0 \sqrt{\text{child}}
\]

<table>
<thead>
<tr>
<th>Without <strong>H</strong></th>
<th>With <strong>H</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>child {a, b, c}</td>
<td>{\bigcup A \mid A \subseteq {x^\dag \mid x \in {a, b, c}}}</td>
</tr>
</tbody>
</table>
The $\mathit{h}$-shifter can be applied to the $nP$ of any noun. In singular/numeral-modified nouns, $[\mathit{sg}] / \mathit{card}$ rules out most GQs except a few Boolean disjunctions.

(30)  a.  child

(31) Which book/books does John have to read?

(32) Which (two) children formed a team?
The $h$-shifter can be applied to the $nP$ of any noun. In singular/numeral-modified nouns, $[sg]/card$ rules out most GQs except a few Boolean disjunctions.

(30) a. child

\[
\phi_P \\
\phi^0 \quad (H) \quad nP \\
[sG] \quad n^0 \quad \sqrt{child}
\]

b. two children

\[
\phi_P \\
\phi^0 \\
[PL] \quad NumP \\
Num^0 \quad Num' \\
two \quad Num^0 \quad (H) \quad nP \\
CARD \quad \ldots
\]

Without $h$

\[
\begin{array}{l}
\text{child} \\
\{a, b, c\}
\end{array}
\]

\[
\begin{array}{l}
\text{two children} \\
\{a \oplus b, b \oplus c, a \oplus c\}
\end{array}
\]

With $h$

\[
\begin{array}{l}
\text{child} \\
\{\bigcup A \mid A \subseteq \{x^\dagger \mid x \in \{a, b, c\}\}\}
\end{array}
\]

\[
\begin{array}{l}
\text{two children} \\
\{\bigcup A \mid A \subseteq \{x^\dagger \mid x \in \{a \oplus b, b \oplus c, a \oplus c\}\}\}
\end{array}
\]

HO-readings are derived uniformly: (i) applying an $h$-shifter to the $nP$ within the $wh$-complement yields a HO-domain; (ii) the $whP$ binds a HO-trace.

(31) Which book/books does John have to read?

\[
[[ \text{which} [sg]/[pl]^h \text{book} ] \lambda \pi_{(et,t)} [ \text{have-to} [ \pi \lambda x_e [ J \text{read} x ] ]]]
\]

(32) Which (two) children formed a team?

\[
[[ \text{which} [pl]-\text{(two)}^h \text{child} ] \lambda \pi_{(et,t)} [ \pi \lambda x_e [ x \text{formed a team} ]]]
\]
(33) Which textbook can we use?

a. \( \pi \gg \Diamond \) (without \( \text{dou} \)): \text{conj-MA}

\[
\begin{array}{c}
\text{CP} \\
\quad \vdots \\
\quad \text{IP} \\
\quad \pi_{(et,t)} \\
\quad \lambda x \quad \Diamond \text{Of}(x_e)
\end{array}
\]

\[
\begin{array}{c}
\text{[cp[ which [sg]-HTEXTBOOK ] \( \lambda \pi_{(et,t)} \) [IP ... }
\end{array}
\]

\[
\begin{array}{c}
\Diamond \text{Of}(a) \land \Diamond \text{Of}(b) \\
\Diamond \text{Of}(a) \lor \Diamond \text{Of}(b)
\end{array}
\]
(33) Which textbook can we use?

a. $\pi \gg \Diamond$ (without $\text{dou}$): $\text{conj-MA}$

```
CP
  ...
  IP
  $\pi_{(et,t)}$
  $\lambda x \Diamond \text{Of}(x_e)$
```

b. $\pi \gg \Diamond$ (with $\text{dou}$): $\text{conj/disj-MA}$

```
CP
  ...
  IP
  $\text{dou}$
  $\pi_{(et,t)}$
  $\lambda x \Diamond \text{Of}(x_e)$
```

```
[CP[ which $\text{[sg]^{H TEXTBOOK}} ] \lambda \pi_{(et,t)} [IP ...}
```

```
\Diamond \text{Of}(a) \land \Diamond \text{Of}(b)

\Diamond \text{Of}(a) \lor \Diamond \text{Of}(b)
```

```
\text{dou}[\Diamond \text{Of}(a) \land \Diamond \text{Of}(b)]

\text{dou}[\Diamond \text{Of}(a) \lor \Diamond \text{Of}(b)]
```

```
\text{dou}[\Diamond \text{Of}(a) \land \Diamond \text{Of}(b)]

\text{dou}[\Diamond \text{Of}(a) \lor \Diamond \text{Of}(b)]
```

```
\text{dou}[\Diamond \text{Of}(a) \land \Diamond \text{Of}(b)]

\text{dou}[\Diamond \text{Of}(a) \lor \Diamond \text{Of}(b)]
```
(33) Which textbook can we use?

a. $\pi \gg \Diamond$ (without $\text{dou}$): $\text{conj-MA}$

\[
\begin{array}{c}
\text{CP} \\
\quad \text{IP} \\
\quad \pi_{(et,t)} \\
\lambda x \Diamond \text{Of}(x_e)
\end{array}
\]

b. $\pi \gg \Diamond$ (with $\text{dou}$): $\text{conj-/disj-MA}$

\[
\begin{array}{c}
\text{CP} \\
\quad \text{IP} \\
\quad \text{DOU} \\
\text{DOU}$ \pi_{(et,t)} \\
\lambda x \Diamond \text{Of}(x_e)
\end{array}
\]

c. $\Diamond \gg \pi$ (with $\text{dou}$): $\text{disj-MA}$

\[
\begin{array}{c}
\text{CP} \\
\quad \text{IP} \\
\quad \text{DOU} \\
\Diamond \text{DOU}$ \pi_{(et,t)} \\
\lambda x \Diamond \text{Of}(x_e)
\end{array}
\]
• Derivation
  – Applying the *-shifter to the *P within the *-complement returns a *-restrctor denoting a set of HO-meanings.
  – The fronted *P binds a HO-trace in the question nucleus.
• **Derivation**
  
  – Applying the n shifter to the nP within the wh-complement returns a *wh*-restrictor denoting a set of HO-meanings.
  – The fronted *whP* binds a HO-trace in the question nucleus.

![Diagram](attachment:diagram.png)

• **Distribution** (updated):  
  *Any whP may license a HO-readings.* If the *wh*-complement is singular or numeral-modified, the atomicity/cardinality requirement of [sc]/[card] rules out most elements in \( ^{h}nP \) except a few disjunctions, causing a disj–conj asymmetry.
Part D. Constraints on HO-answers

Part A: the Q-domain yielded by a semantically unmarked whP contains at least conj-/disj-unctions over \([wh\text{-complement}]\) as well as their Boolean coordinations.
Part D. Constraints on HO-answers

Part A: the Q-domain yielded by a semantically unmarked \(wh\)P contains at least conj-/disj-unctions over \([wh\)-complement]\) as well as their Boolean coordinations.

Next: Can we make the following generalization?

The Q-domain yielded by a (semantically unmarked) \(wh\)P consists of all GQs ranging over a subset of \([wh\)-complement]\) and the Boolean combinations of these GQs.
Part D. Constraints on HO-answers

**Part A:** the Q-domain yielded by a semantically unmarked $whP$ contains at least conj-/disj-unctions over $[wh$-complement] as well as their Boolean coordinations.

**Next:** Can we make the following generalization?

The Q-domain yielded by a (semantically unmarked) $whP$ consists of all GQs ranging over a subset of $[wh$-complement] and the Boolean combinations of these GQs.

In other words, are there any GQ(-compound)s that cannot be included in a Q-domain?
Completeness-based Tests

The completeness condition of question-embedding

‘x knows Q.’ \iff ‘x knows the/a complete true answer of Q.’
### Completeness-based Tests

<table>
<thead>
<tr>
<th>The completeness condition of question-embedding</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘x knows Q.’ $\sim$ ‘x knows the/a complete true answer of Q.’</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Completeness-based Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(generalized from Spector 2008)</td>
</tr>
<tr>
<td>For any proposition $p$ that names a short answer $\alpha$ to an (exhaustive) question $Q$, if there is a world in which both $p$ and ‘$x$ knows $Q$’ are true but ‘$x$ knows $p$’ isn’t true, then: $p \notin$ Hamblin set, and $\alpha \notin Q$-domain.</td>
</tr>
</tbody>
</table>
Completeness-based Tests

The completeness condition of question-embedding

‘x knows Q.’ ⇔ ‘x knows the/a complete true answer of Q.’

The Completeness-based Test (generalized from Spector 2008)

For any proposition p that names a short answer α to an (exhaustive) question Q, if there is a world in which both p and ‘x knows Q’ are true but ‘x knows p’ isn’t true, then:

- p ∉ Hamblin set, and
- α ∉ Q-domain.

Example

John’s summer reading obligations include the following:

a. he has to read at least two French novels;

b. he has to read no Russian novel (since he has read too many Russian novels).

(34) ‘Sue knows which books John has to read this summer.’
Completeness-based Tests

**The completeness condition of question-embedding**

‘x knows Q.’ ⇾ ‘x knows the/a complete true answer of Q.’

**The Completeness-based Test** (generalized from Spector 2008)

For any proposition \( p \) that names a short answer \( \alpha \) to an (exhaustive) question \( Q \), if there is a world in which both \( p \) and ‘\( x \) knows \( Q \)’ are true but ‘\( x \) knows \( p \)’ isn’t true, then: \( p \notin \text{Hamblin set} \), and \( \alpha \notin \text{Q-domain} \).

**Example**

John’s summer reading obligations include the following:

a. he has to read at least two French novels;
b. he has to read no Russian novel (since he has read too many Russian novels).

(34) ‘Sue knows which books John has to read this summer.’

\( \sim \) Sue knows (a).
Completeness-based Tests

The completeness condition of question-embedding

‘x knows Q.’ ⇾ ‘x knows the/a complete true answer of Q.’

The Completeness-based Test

(generalized from Spector 2008)

For any proposition $p$ that names a short answer $\alpha$ to an (exhaustive) question Q, if there is a world in which both $p$ and ‘x knows Q’ are true but ‘x knows p’ isn’t true, then: $p \notin$ Hamblin set, and $\alpha \notin$ Q-domain.

Example

<table>
<thead>
<tr>
<th>John’s summer reading obligations include the following:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. he has to read at least two French novels;</td>
</tr>
<tr>
<td>b. he has to read no Russian novel (since he has read too many Russian novels).</td>
</tr>
</tbody>
</table>

(34) ‘Sue knows which books John has to read this summer.’

$\leadsto$ Sue knows (a). increasing
$\not\leadsto$ Sue knows (b). decreasing
Completeness-based Tests

The completeness condition of question-embedding

‘x knows Q.’ ⇝ ‘x knows the/a complete true answer of Q.’

The Completeness-based Test (generalized from Spector 2008)

For any proposition \( p \) that names a short answer \( \alpha \) to an (exhaustive) question \( Q \), if there is a world in which both \( p \) and ‘x knows Q’ are true but ‘x knows \( p \)’ isn’t true, then: \( p \notin \) Hamblin set, and \( \alpha \notin \) Q-domain.

Example

John’s summer reading obligations include the following:

a. he has to read at least two French novels;
b. he has to read no Russian novel (since he has read too many Russian novels).

(34) ‘Sue knows which books John has to read this summer.’

\( \rightsquigarrow \) Sue knows (a).  
increasing

\( \triangleright \) Sue knows (b).  
decreasing

\( \leftrightarrow \) Sue knows (a)-and-(b).  
non-monotonic
Increasing-GQ Constraint (Spector 2008)

The HO-meanings in the Q-domain of a *wh*-question must be **increasing**.
The increasing-GQ constraint

**Increasing-GQ Constraint** *(Spector 2008)*

The HO-meanings in the Q-domain of a *wh*-question must be **increasing**.

**Example**

John’s summer reading obligations include the following:

- a. he has to read **at least two** French novels;
- b. he has to read **no** Russian novel;

(35) ‘Sue knows which books John has to read this summer.’

\( \neg \forall \text{Sue knows (a)-and-(b).} \)  
complex non-monotonic GQ
The increasing-GQ constraint

**Increasing-GQ Constraint (Spector 2008)**

The HO-meanings in the Q-domain of a *wh*-question must be **increasing**.

**Example**

John’s summer reading obligations include the following:

- a. he has to read **at least two French novels**;
- b. he has to read **no Russian novel**;
- c. he has to read **exactly two Chinese novels**.

(35) ‘Sue knows which books John has to read this summer.’

\( \not\models \text{Sue knows } (a)\text{-and-}(b). \)

complex non-monotonic GQ
The increasing-GQ constraint

**Increasing-GQ Constraint**  
(Spector 2008)

The HO-meanings in the Q-domain of a *wh*-question must be **increasing**.

**Example**

John’s summer reading obligations include the following:

a. he has to read **at least two French novels**;
b. he has to read **no Russian novel**;
c. he has to read **exactly two Chinese novels**.

(35) ‘Sue knows which books John has to read this summer.’

\[
\leftrightarrow \text{Sue knows} \ (a)\text{-and-} (b). \\
\sim \text{Sue knows} \ (c).
\]

- complex non-monotonic GQ
- simplex non-monotonic GQ
The increasing-GQ constraint

**Increasing-GQ Constraint** (Spector 2008)

The HO-meanings in the Q-domain of a *wh*-question must be **increasing**.

**Example**

John’s summer reading obligations include the following:

a. he has to read **at least two French novels**;
b. he has to read **no Russian novel**;
c. he has to read **exactly two Chinese novels**.

(35) ‘Sue knows which books John has to read this summer.’

\[ \iff \text{Sue knows (a)-and-(b).} \text{ complex non-monotonic GQ} \]

\[ \leadsto \text{Sue knows (c).} \text{ simplex non-monotonic GQ} \]

⇒ Spector’s Increasing-GQ requirement is too strong.
The increasing-GQ constraint

**Increasing-GQ Constraint**  
(Spector 2008)

The HO-meanings in the Q-domain of a *wh*-question must be **increasing**.

**Example**

John’s summer reading obligations include the following:

a. he has to read **at least two French novels**;

b. he has to read **no Russian novel**;

c. he has to read **exactly two Chinese novels**.

(35) ‘Sue knows which books John has to read this summer.’

\[ \forall S \text{ Sue knows } (a) \text{-and-} (b). \]

complex non-monotonic GQ

\[ \exists S \text{ Sue knows } (c). \]

simplex non-monotonic GQ

Spector’s Increasing-GQ requirement is too strong. Any monotonicity-based constraint faces a dilemma in differentiating between *(a)-and-*(b) and *(c).*
The increasing-GQ constraint

**Increasing-GQ Constraint** (Spector 2008)

The HO-meanings in the Q-domain of a *wh*-question must be **increasing**.

**Example**

John’s summer reading obligations include the following:

a. he has to read **at least two French novels**;

b. he has to read **no Russian novel**;

c. he has to read **exactly two Chinese novels**.

(35) ‘Sue knows which books John has to read this summer.’

\[ \not \supset \text{Sue knows (a)-and-(b).} \]

\[ \supset \text{Sue knows (c).} \]

\[ \supset \text{Spector’s Increasing-GQ requirement is too strong. Any monotonicity-based constraint faces a dilemma in differentiating between (a)-and-(b) and (c).} \]

\[ \not \supset \text{Spector’s Increasing-GQ requirement is too strong. Any monotonicity-based constraint faces a dilemma in differentiating between (a)-and-(b) and (c).} \]

**NB:** It isn’t about differentiating between simplex and complex non-monotonic GQs.

\[ \supset \text{at least three French novels but no more than 20 novels (of any kind)}. \]

\[ \not \supset \text{less than three or more than ten novels}. \]
Claim: Whether a GQ can be included in a Q-domain is determined by its *positiveness*. 
The Positive-GQ Constraint

Claim: Whether a GQ can be included in a Q-domain is determined by its positiveness. A GQ being positive means that it ensures existence w.r.t. the set it ranges over.

For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is positive iff $\pi \subseteq E(smlo(\pi))$.
(E.g.: exactly two books $\subseteq$ some books; every book or no book $\not\subseteq$ some book.)
Claim: Whether a GQ can be included in a Q-domain is determined by its \textbf{positiveness}. A GQ being positive means that it ensures \textbf{existence} w.r.t. the set it ranges over.

For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is \textbf{positive} iff $\pi \subseteq \mathbb{E} (\text{SMLO}(\pi))$.

(E.g.: \textit{exactly two books} $\subseteq$ \textit{some books}; \textit{every book or no book} $\not\subseteq$ \textit{some book}.)

<table>
<thead>
<tr>
<th>GQ $\pi$</th>
<th>SMLO($\pi$)</th>
<th>Increasing? $\uparrow_{\text{MON}}$</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \uparrow$</td>
<td>${a}$</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$a \uparrow \cap b \uparrow$, $a \uparrow \cup b \uparrow$</td>
<td>${a,b}$</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>{at least, more than} two books</td>
<td>books</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>every book except $a$</td>
<td>$\text{book} - {a}$</td>
<td>Yes $\uparrow_{\text{MON}}$</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Claim: Whether a GQ can be included in a Q-domain is determined by its **positiveness**. A GQ being positive means that it ensures **existence** w.r.t. the set it ranges over.

For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is **positive** iff $\pi \subseteq E(smlo(\pi))$.

(E.g.: *exactly two books* $\subseteq$ *some books*; *every book or no book* $\not\subseteq$ *some book.*

<table>
<thead>
<tr>
<th>GQ $\pi$</th>
<th>$smlo(\pi)$</th>
<th>Increasing?</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^\uparrow$</td>
<td>${a}$</td>
<td>Yes ($\uparrow_{MON}$)</td>
<td>Yes</td>
</tr>
<tr>
<td>$a^\uparrow \cap b^\uparrow, a^\uparrow \cup b^\uparrow$</td>
<td>${a, b}$</td>
<td>Yes ($\uparrow_{MON}$)</td>
<td>Yes</td>
</tr>
<tr>
<td>${at least, more than}$ two books</td>
<td>$books$</td>
<td>Yes ($\uparrow_{MON}$)</td>
<td>Yes</td>
</tr>
<tr>
<td>every book except $a$</td>
<td>$book - {a}$</td>
<td>Yes ($\uparrow_{MON}$)</td>
<td>Yes</td>
</tr>
<tr>
<td>${at most, less than, no more than}$ two books</td>
<td>$books$</td>
<td>No ($\downarrow_{MON}$)</td>
<td>No</td>
</tr>
<tr>
<td>no book except $a$</td>
<td>$book - {a}$</td>
<td>No ($\downarrow_{MON}$)</td>
<td>No</td>
</tr>
</tbody>
</table>
Claim: Whether a GQ can be included in a Q-domain is determined by its positiveness. A GQ being positive means that it ensures existence w.r.t. the set it ranges over.

For any $\pi$ of type $\langle et, t \rangle$, $\pi$ is positive iff $\pi \subseteq E(\text{smlo}(\pi))$.
(E.g.: exactly two books $\subseteq$ some books; every book or no book $\not\subseteq$ some book.)

<table>
<thead>
<tr>
<th>GQ $\pi$</th>
<th>smlo(\pi)</th>
<th>Increasing?</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^{↑}$</td>
<td>${a}$</td>
<td>Yes ($\uparrow_{\text{MON}}$)</td>
<td>Yes</td>
</tr>
<tr>
<td>$a^{↑} \cap b^{↑}, a^{↑} \cup b^{↑}$</td>
<td>${a, b}$</td>
<td>Yes ($\uparrow_{\text{MON}}$)</td>
<td>Yes</td>
</tr>
<tr>
<td>{at least, more than} two books</td>
<td>books</td>
<td>Yes ($\uparrow_{\text{MON}}$)</td>
<td>Yes</td>
</tr>
<tr>
<td>every book except $a$</td>
<td>$\text{book} - {a}$</td>
<td>Yes ($\uparrow_{\text{MON}}$)</td>
<td>Yes</td>
</tr>
<tr>
<td>{at most, less than, no more than} two books</td>
<td>books</td>
<td>No ($\downarrow_{\text{MON}}$)</td>
<td>No</td>
</tr>
<tr>
<td>no book except $a$</td>
<td>$\text{book} - {a}$</td>
<td>No ($\downarrow_{\text{MON}}$)</td>
<td>No</td>
</tr>
<tr>
<td>less than three or more than ten books</td>
<td>books</td>
<td>No (n.m.)</td>
<td>No</td>
</tr>
<tr>
<td>every or no book</td>
<td>books</td>
<td>No (n.m.)</td>
<td>No</td>
</tr>
<tr>
<td>exactly two books</td>
<td>books</td>
<td>No (n.m.)</td>
<td>Yes</td>
</tr>
<tr>
<td>two to four books</td>
<td>books</td>
<td>No (n.m.)</td>
<td>Yes</td>
</tr>
<tr>
<td>some but not all books</td>
<td>books</td>
<td>No (n.m.)</td>
<td>Yes</td>
</tr>
<tr>
<td>(exactly) two or four books</td>
<td>books</td>
<td>No (n.m.)</td>
<td>Yes</td>
</tr>
<tr>
<td>an even number of books</td>
<td>books</td>
<td>No (n.m.)</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 1: Monotonicity versus positiveness
The Positive-GQ Constraint

Claim: Whether a GQ can be included in a Q-domain is determined by its positivity. A GQ being positive means that it ensures existence w.r.t. the set it ranges over.

For any \( \pi \) of type \( \langle et, t \rangle \), \( \pi \) is positive iff \( \pi \subseteq \mathbb{E}(\text{smlo}(\pi)) \).

(E.g.: exactly two books \( \subseteq \) some books; every book or no book \( \not\subseteq \) some book.)

<table>
<thead>
<tr>
<th>GQ ( \pi )</th>
<th>smlo(( \pi ))</th>
<th>Increasing?</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \uparrow )</td>
<td>{a}</td>
<td>Yes (( \uparrow_{\text{mon}} ))</td>
<td>Yes</td>
</tr>
<tr>
<td>( a \uparrow \cap b \uparrow, a \uparrow \cup b \uparrow )</td>
<td>{a, b}</td>
<td>Yes (( \uparrow_{\text{mon}} ))</td>
<td>Yes</td>
</tr>
<tr>
<td>{at least, more than} two books</td>
<td>books</td>
<td>Yes (( \uparrow_{\text{mon}} ))</td>
<td>Yes</td>
</tr>
<tr>
<td>every book except ( a )</td>
<td>\text{book} \setminus {a}</td>
<td>Yes (( \uparrow_{\text{mon}} ))</td>
<td>Yes</td>
</tr>
<tr>
<td>{at most, less than, no more than} two books</td>
<td>books</td>
<td>No (( \downarrow_{\text{mon}} ))</td>
<td>No</td>
</tr>
<tr>
<td>no book except ( a )</td>
<td>\text{book} \setminus {a}</td>
<td>No (( \downarrow_{\text{mon}} ))</td>
<td>No</td>
</tr>
<tr>
<td>less than three or more than ten books</td>
<td>books</td>
<td>No (N.M.)</td>
<td>No</td>
</tr>
<tr>
<td>every or no book</td>
<td>books</td>
<td>No (N.M.)</td>
<td>No</td>
</tr>
<tr>
<td>exactly two books</td>
<td>books</td>
<td>No (N.M.)</td>
<td>Yes</td>
</tr>
<tr>
<td>two to four books</td>
<td>books</td>
<td>No (N.M.)</td>
<td>Yes</td>
</tr>
<tr>
<td>some but not all books</td>
<td>books</td>
<td>No (N.M.)</td>
<td>Yes</td>
</tr>
<tr>
<td>(exactly) two or four books</td>
<td>books</td>
<td>No (N.M.)</td>
<td>Yes</td>
</tr>
<tr>
<td>an even number of books</td>
<td>books</td>
<td>No (N.M.)</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 1: Monotonicity versus positiveness

Positive-GQ Constraint (preliminary)

GQs in the Q-domain of a \( wh \)-question must be positive.
The Homo-Positive-GQ Constraint

<table>
<thead>
<tr>
<th>GQ ( \pi )</th>
<th>( \text{smlo}(\pi) )</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ <em>at least two</em> ( A )</td>
<td>( A )</td>
<td>Yes</td>
</tr>
<tr>
<td>✓ <em>exactly two</em> ( A )</td>
<td>( A )</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Benjamin Spector (p.c.): Positiveness does not exclude many unwanted non-monotonic GQ-coordinations. (E.g., *every article and no book*).

<table>
<thead>
<tr>
<th>GQ $\pi$</th>
<th>$\text{smlo}(\pi)$</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ <em>at least two A</em></td>
<td>$A$</td>
<td>Yes</td>
</tr>
<tr>
<td>✓ <em>exactly two A</em></td>
<td>$A$</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Benjamin Spector (p.c.): Positiveness does not exclude many unwanted non-monotonic GQ-coordinations. (E.g., *every article and no book*).

<table>
<thead>
<tr>
<th>GQ $\pi$</th>
<th>$\text{smlo}(\pi)$</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ at least two $A$</td>
<td>$A$</td>
<td>Yes</td>
</tr>
<tr>
<td>✓ exactly two $A$</td>
<td>$A$</td>
<td>Yes</td>
</tr>
<tr>
<td>✗ every $A$ and no $B$</td>
<td>$A \cup B$</td>
<td>Yes</td>
</tr>
</tbody>
</table>
The Homo-Positive-GQ Constraint

Benjamin Spector (p.c.): Positiveness does not exclude many unwanted non-monotonic GQ-coordinations. (E.g., *every article and no book*).

<table>
<thead>
<tr>
<th>GQ $\pi$</th>
<th>$\text{smlo}(\pi)$</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>✔ at least two $A$</td>
<td>$A$</td>
<td>Yes</td>
</tr>
<tr>
<td>✔ exactly two $A$</td>
<td>$A$</td>
<td>Yes</td>
</tr>
<tr>
<td>✗ every $A$ and no $B$</td>
<td>$A \cup B$</td>
<td>Yes</td>
</tr>
</tbody>
</table>

We need to examine whether existence is ensured *homogeneously* w.r.t. the range-over set of the coordinated *increasing* GQ as well as that of the coordinated *decreasing* GQ.
The Homo-Positive-GQ Constraint

Benjamin Spector (p.c.): Positiveness does not exclude many unwanted non-monotonic GQ-coordinations. (E.g., *every article and no book*).

<table>
<thead>
<tr>
<th>GQ π</th>
<th>smlo(π)</th>
<th>Positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ at least two A</td>
<td>A</td>
<td>Yes</td>
</tr>
<tr>
<td>✓ exactly two A</td>
<td>A</td>
<td>Yes</td>
</tr>
<tr>
<td>X every A and no B</td>
<td>A ∪ B</td>
<td>Yes</td>
</tr>
</tbody>
</table>

We need to examine whether existence is ensured *homogeneously* w.r.t. the range-over set of the coordinated *increasing* GQ as well as that of the coordinated *decreasing* GQ.

For any $\pi \in D_{(et,t)}$, $\pi$ is **homo-positive** iff $\pi \subseteq \mathbb{E}(\text{smlo}(\pi^+))$ and $\pi \subseteq \mathbb{E}(\text{smlo}(\pi^-))$.

- $\pi^+ =_{df} \{ P \mid \exists P' \subseteq P[\pi(P')] \}$: the strongest increasing GQ entailed by $\pi$;
- $\pi^- =_{df} \{ P \mid \exists P' \supseteq P[\pi(P')] \}$: the strongest decreasing GQ entailed by $\pi$;
Benjamin Spector (p.c.): Positiveness does not exclude many unwanted non-monotonic GQ-coordinations. (E.g., *every article and no book*).

<table>
<thead>
<tr>
<th>GQ $\pi$</th>
<th>$\text{smlo}(\pi)$</th>
<th>Positive?</th>
<th>$\text{smlo}(\pi^+)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>✔ at least two $A$</td>
<td>$A$</td>
<td>Yes</td>
<td>$A$</td>
</tr>
<tr>
<td>✔ exactly two $A$</td>
<td>$A$</td>
<td>Yes</td>
<td>$A$</td>
</tr>
<tr>
<td>✗ every $A$ and no $B$</td>
<td>$A \cup B$</td>
<td>Yes</td>
<td>$A$</td>
</tr>
</tbody>
</table>

We need to examine whether existence is ensured **homogeneously** w.r.t. the range-over set of the coordinated increasing GQ as well as that of the coordinated decreasing GQ.

For any $\pi \in D_{(et,t)}$, $\pi$ is **homo-positive** iff $\pi \subseteq E(\text{smlo}(\pi^+))$ and $\pi \subseteq E(\text{smlo}(\pi^-))$.

- $\pi^+ = \text{df} \{ P \mid \exists P' \subseteq P[\pi(P')] \}$ the strongest increasing GQ entailed by $\pi$;
- $\pi^- = \text{df} \{ P \mid \exists P' \supseteq P[\pi(P')] \}$ the strongest decreasing GQ entailed by $\pi$;
The Homo-Positive-GQ Constraint

Benjamin Spector (p.c.): Positiveness does not exclude many unwanted non-monotonic GQ-coordinations. (E.g., *every article and no book*).

<table>
<thead>
<tr>
<th>GQ ( \pi )</th>
<th>SMLO(( \pi ))</th>
<th>Positive?</th>
<th>SMLO(( \pi^+ ))</th>
<th>SMLO(( \pi^- ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>✔️ at least two A</td>
<td>A</td>
<td>Yes</td>
<td>A</td>
<td>( D_e )</td>
</tr>
<tr>
<td>✔️ exactly two A</td>
<td>A</td>
<td>Yes</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>✗ every A and no B</td>
<td>A ∪ B</td>
<td>Yes</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

We need to examine whether existence is ensured **homogeneously** w.r.t. the range-over set of the coordinated increasing GQ as well as that of the coordinated decreasing GQ.

For any \( \pi \in D_{(et,t)} \), \( \pi \) is **homo-positive** iff \( \pi \subseteq \mathbb{E}(\text{SMLO}(\pi^+)) \) and \( \pi \subseteq \mathbb{E}(\text{SMLO}(\pi^-)) \).

- \( \pi^+ =_{df} \{ P \mid \exists P' \subseteq P[\pi(P')] \} \) the strongest increasing GQ entailed by \( \pi \);  
- \( \pi^- =_{df} \{ P \mid \exists P' \supseteq P[\pi(P')] \} \) the strongest decreasing GQ entailed by \( \pi \);
Benjamin Spector (p.c.): Positiveness does not exclude many unwanted non-monotonic GQ-coordinations. (E.g., every article and no book).

We need to examine whether existence is ensured homogeneously w.r.t. the range-over set of the coordinated increasing GQ as well as that of the coordinated decreasing GQ.

For any $\pi \in D_{(et,t)}$, $\pi$ is homo-positive iff $\pi \subseteq E(smlo(\pi^+))$ and $\pi \subseteq E(smlo(\pi^-))$.

- $\pi^+ =_{df} \{ P \mid \exists P' \subseteq P[\pi(P')] \}$ the strongest increasing GQ entailed by $\pi$;
- $\pi^- =_{df} \{ P \mid \exists P' \supseteq P[\pi(P')] \}$ the strongest decreasing GQ entailed by $\pi$;
Benjamin Spector (p.c.): Positiveness does not exclude many unwanted non-monotonic GQ-coordinations. (E.g., every article and no book).

<table>
<thead>
<tr>
<th>GQ $\pi$</th>
<th>$\text{smlo}(\pi)$</th>
<th>Positive?</th>
<th>$\text{smlo}(\pi^+)$</th>
<th>$\text{smlo}(\pi^-)$</th>
<th>Homo-positive?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\checkmark$ at least two $A$</td>
<td>$A$</td>
<td>Yes</td>
<td>$A$</td>
<td>$D_e$</td>
<td>Yes</td>
</tr>
<tr>
<td>$\checkmark$ exactly two $A$</td>
<td>$A$</td>
<td>Yes</td>
<td>$A$</td>
<td>$A$</td>
<td>Yes</td>
</tr>
<tr>
<td>$\times$ every $A$ and no $B$</td>
<td>$A \cup B$</td>
<td>Yes</td>
<td>$A$</td>
<td>$B$</td>
<td>No</td>
</tr>
</tbody>
</table>

We need to examine whether existence is ensured **homogeneously** w.r.t. the range-over set of the coordinated increasing GQ as well as that of the coordinated decreasing GQ.

For any $\pi \in D_{(et,t)}$, $\pi$ is **homo-positive** iff $\pi \subseteq \mathbb{E}(\text{smlo}(\pi^+))$ and $\pi \subseteq \mathbb{E}(\text{smlo}(\pi^-))$.

- $\pi^+ =_{df} \{ P | \exists P' \subseteq P[\pi(P')] \}$ the strongest increasing GQ entailed by $\pi$;
- $\pi^- =_{df} \{ P | \exists P' \supseteq P[\pi(P')] \}$ the strongest decreasing GQ entailed by $\pi$;

**Homo-Positive-GQ Constraint (final)**

GQs in the Q-domain of a $wh$-question must be **homo-positive**.
(A, B, C are three sets of entities; C is a superset of A, and B is not.)

<table>
<thead>
<tr>
<th>GQ π</th>
<th>π⁺</th>
<th>π⁻</th>
<th>Mon?</th>
<th>Pos?</th>
<th>Homo-Pos?</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least 2 A</td>
<td>at least 2 A</td>
<td>(D_{(e,t)})</td>
<td>↑\text{MON}</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>an even number of A</td>
<td>at least 2 A</td>
<td>(D_{(e,t)})</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 A</td>
<td>≥ 2 A</td>
<td>≤ 2 A</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 to 4 A</td>
<td>≥ 2 A</td>
<td>≤ 4 A</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 or 4 A</td>
<td>≥ 2 A</td>
<td>≤ 4 A</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>no more than 4 A</td>
<td>(D_{(e,t)})</td>
<td>≤ 4 A</td>
<td>↓\text{MON}</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>less than 2 or more than 5 A</td>
<td>(D_{(e,t)})</td>
<td>(D_{(e,t)})</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>every or no A</td>
<td>every A</td>
<td>no A</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
(A, B, C are three sets of entities; C is a superset of A, and B is not.)

<table>
<thead>
<tr>
<th>GQ π</th>
<th>π⁺</th>
<th>π⁻</th>
<th>Mon?</th>
<th>Pos?</th>
<th>Homo-Pos?</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least 2 A</td>
<td>at least 2 A</td>
<td>D_{e,t}</td>
<td>↑_{MON}</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>an even number of A</td>
<td>at least 2 A</td>
<td>D_{e,t}</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 A</td>
<td>≥ 2 A</td>
<td>≤ 2 A</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 to 4 A</td>
<td>≥ 2 A</td>
<td>≤ 4 A</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 or 4 A</td>
<td>≥ 2 A</td>
<td>≤ 4 A</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>no more than 4 A</td>
<td>D_{e,t}</td>
<td>≤ 4 A</td>
<td>↓_{MON}</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>less than 2 or more than 5 A</td>
<td>D_{e,t}</td>
<td>D_{e,t}</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>every or no A</td>
<td>every A</td>
<td>no A</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>every A or no B</td>
<td>every A</td>
<td>no B</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>every A or no C</td>
<td>every A</td>
<td>no C</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
(A, B, C are three sets of entities; C is a superset of A, and B is not.)

<table>
<thead>
<tr>
<th>GQ π</th>
<th>(\pi^+)</th>
<th>(\pi^-)</th>
<th>Mon?</th>
<th>Pos?</th>
<th>Homo-Pos?</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least 2 A</td>
<td>at least 2 A</td>
<td>(D_{(\epsilon,t)})</td>
<td>(\uparrow_{\text{MON}})</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>an even number of A</td>
<td>at least 2 A</td>
<td>(D_{(\epsilon,t)})</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 A</td>
<td>(\geq 2 A)</td>
<td>(\leq 2 A)</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 to 4 A</td>
<td>(\geq 2 A)</td>
<td>(\leq 4 A)</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>exactly 2 or 4 A</td>
<td>(\geq 2 A)</td>
<td>(\leq 4 A)</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>no more than 4 A</td>
<td>(D_{(\epsilon,t)})</td>
<td>(\leq 4 A)</td>
<td>(\downarrow_{\text{MON}})</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>less than 2 or more than 5 A</td>
<td>(D_{(\epsilon,t)})</td>
<td>(D_{(\epsilon,t)})</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>every or no A</td>
<td>every A</td>
<td>no A</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>every A or no B</td>
<td>every A</td>
<td>no B</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>every A or no C</td>
<td>every A</td>
<td>no C</td>
<td>N.M.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>every A but no B</td>
<td>every A</td>
<td>no B</td>
<td>N.M.</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>at least 2 A but (\leq 4 B)</td>
<td>(\geq 2 A)</td>
<td>(\leq 4 B)</td>
<td>N.M.</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>at least 2 A but (\leq 4 C)</td>
<td>(\geq 2 A)</td>
<td>(\leq 4 C)</td>
<td>N.M.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 2: Monotonicity (Mon) versus positiveness (Pos) versus homo-positiveness
• Not all GQs can be used as semantic answers to *wh*-questions.
Interim summary

- Not all GQs can be used as semantic answers to *wh*-questions.
  - In general, increasing GQs are qualified while decreasing GQs are not.
  - There are subtle variations among the non-monotonic GQs. I account for these variations in terms of homo-positiveness.
Interim summary

- Not all GQs can be used as semantic answers to *wh*-questions.
  - In general, increasing GQs are qualified while decreasing GQs are not.
  - There are subtle variations among the non-monotonic GQs. I account for these variations in terms of homo-positiveness.

- Where does homo-positiveness come from?
• Not all GQs can be used as semantic answers to \textit{wh}-questions.
  • In general, increasing GQs are qualified while decreasing GQs are not.
  • There are subtle variations among the non-monotonic GQs. I account for these variations in terms of homo-positiveness.

• Where does homo-positiveness come from? Unclear yet.
  • lexicon of the $h$-shifter
  • (existential) presupposition of the HO-trace
  • constraint on semantic reconstruction
  • matter of pragmatics
  • ...
Conclusions

• Evidence from questions with □-modals or stubbornly collective predicates argues that sometimes a wh-question must be interpreted with a HO-reading.

• Derivation HO-readings arise if the /h.sc-shifter converts the wh-restrictor into a set of GQs and the wh-Pbinds to HO-trace. Hence, HO-readings are unavailable if the use of /h.sc is blocked.

• Disj–Conj Asymmetry Puzzlingly, singular and numeral-modified questions admit elided disjunctive answers, although rejecting elided conjunctive answers.

• Uniform approach: Disjunctions (but not conjunctions) may satisfy the atomicity/cardinality constraints of singular/numeral-modified nouns.

• Reconstruction approach (Appendix): Reconstructing the wh-complement yields local uniqueness and further contradictions for conjunctive answers.

• The Homo-Positive-GQ Constraint GQs that can serve as semantic answers to wh-questions must be homo-positive.

Thank you!

This presentation is based on Xiang (2020b) “Higher-order readings of wh-questions”, forthcoming in Natural Language Semantics. For helpful discussions, I thank Luis Alonso-Ovalle, Lucas Champollion, Gennaro Chierchia, Danny Fox, Michael Glanzberg, Manuel Križ, Floris Roelofsen, Vincent Rouillard, Benjamin Spector, Bernhard Schwarz, and the audiences at the Spring 2019 Rutgers Seminar, Georg-August-Universität Göttingen, Ecole Normale Supérieure, and the 22nd Amsterdam Colloquium. I am grateful to two anonymous reviewers of NLS.
Conclusions

• Evidence
  Evidence from questions with □-modals or stubbornly collective predicates argues that sometimes a *wh*-question must be interpreted with a HO-reading.

Thank you!

This presentation is based on Xiang (2020b) “Higher-order readings of *wh*-questions”, forthcoming in *Natural Language Semantics*. For helpful discussions, I thank Luis Alonso-Ovalle, Lucas Champollion, Gennaro Chierchia, Danny Fox, Michael Glanzberg, Manuel Križ, Floris Roelofsen, Vincent Rouillard, Benjamin Spector, Bernhard Schwarz, and the audiences at the Spring 2019 Rutgers Seminar, Georg-August-Universität Göttingen, Ecole Normale Supérieure, and the 22nd Amsterdam Colloquium. I am grateful to two anonymous reviewers of *NLS*.
Conclusions

• **Evidence**
  Evidence from questions with □-modals or stubbornly collective predicates argues that sometimes a *wh*-question must be interpreted with a HO-reading.

• **Derivation**
  HO-readings arise if the *h*-shifter converts the *wh*-restrictor into a set of GQs and the *whP* binds a HO-trace. Hence, HO-readings are unavailable if the use of *h* is blocked.
Conclusions

• **Evidence**
  Evidence from questions with □-modals or stubbornly collective predicates argues that sometimes a *wh*-question must be interpreted with a HO-reading.

• **Derivation**
  HO-readings arise if the *h*-shifter converts the *wh*-restrictor into a set of GQs and the *whP* binds a HO-trace. Hence, HO-readings are unavailable if the use of *h* is blocked.

• **Disj–Conj Asymmetry**
  Puzzlingly, singular and numeral-modified questions admit elided disjunctive answers, although rejecting elided conjunctive answers.
    • *Uniform approach*: Disjunctions (but not conjunctions) may satisfy the atomicity/cardinality constraints of singular/numeral-modified nouns.
    • *Reconstruction approach* (Appendix): Reconstructing the *wh*-complement yields local uniqueness and further contradictions for conjunctive answers.

Thank you!
This presentation is based on Xiang (2020b) “Higher-order readings of *wh*-questions”, forthcoming in *Natural Language Semantics*. For helpful discussions, I thank Luis Alonso-Ovalle, Lucas Champollion, Gennaro Chierchia, Danny Fox, Michael Glanzberg, Manuel Križ, Floris Roelofsen, Vincent Rouillard, Benjamin Spector, Bernhard Schwarz, and the audiences at the Spring 2019 Rutgers Seminar, Georg-August-Universität Göttingen, Ecole Normale Supérieure, and the 22nd Amsterdam Colloquium. I am grateful to two anonymous reviewers of *NLS*. 
Conclusions

• Evidence
Evidence from questions with □-modals or stubbornly collective predicates argues that sometimes a \textit{wh}-question must be interpreted with a HO-reading.

• Derivation
HO-readings arise if the \textit{h}-shifter converts the \textit{wh}-restrictor into a set of GQs and the \textit{whP} binds a HO-trace. Hence, HO-readings are unavailable if the use of \textit{h} is blocked.

• Disj–Conj Asymmetry
Puzzlingly, singular and numeral-modified questions admit elided disjunctive answers, although rejecting elided conjunctive answers.
  
  • \textit{Uniform approach}: Disjunctions (but not conjunctions) may satisfy the atomicity/cardinality constraints of singular/numeral-modified nouns.
  
  • \textit{Reconstruction approach} (Appendix): Reconstructing the \textit{wh}-complement yields local uniqueness and further contradictions for conjunctive answers.

• The Homo-Positive-GQ Constraint
GQs that can serve as semantic answers to \textit{wh}-questions must be homo-positive.
Conclusions

• **Evidence**
  Evidence from questions with □-modals or stubbornly collective predicates argues that sometimes a *wh*-question must be interpreted with a HO-reading.

• **Derivation**
  HO-readings arise if the *h*-shifter converts the *wh*-restricter into a set of GQs and the *whP* binds a HO-trace. Hence, HO-readings are unavailable if the use of *h* is blocked.

• **Disj–Conj Asymmetry**
  Puzzlingly, singular and numeral-modified questions admit elided disjunctive answers, although rejecting elided conjunctive answers.
  - *Uniform approach*: Disjunctions (but not conjunctions) may satisfy the atomicity/cardinality constraints of singular/numeral-modified nouns.
  - *Reconstruction approach* (Appendix): Reconstructing the *wh*-complement yields local uniqueness and further contradictions for conjunctive answers.

• **The Homo-Positive-GQ Constraint**
  GQs that can serve as semantic answers to *wh*-questions must be homo-positive.

Thank you!

This presentation is based on Xiang (2020b) “Higher-order readings of *wh*-questions”, forthcoming in *Natural Language Semantics*. For helpful discussions, I thank Luis Alonso-Ovalle, Lucas Champollion, Gennaro Chierchia, Danny Fox, Michael Glanzberg, Manuel Križ, Floris Roelofsen, Vincent Rouillard, Benjamin Spector, Bernhard Schwarz, and the audiences at the Spring 2019 Rutgers Seminar, Georg-August-Universität Göttingen, Ecole Normale Supérieure, and the 22nd Amsterdam Colloquium. I am grateful to two anonymous reviewers of *NLS*. 
Proposal: The ‘disjunction-only’ HO-reading is an ad hoc reading yielded by *syntactically reconstructing* the *wh*-complement to the nucleus. This reconstruction ...

- ... leaves an unmarked domain variable $D$, which can host the $h$-shifter.
- ... yields *uniqueness* in the Q-nucleus. Crucially, conjoining two uniqueness statements results in a contradiction.

Roughly:

(36) Which book does John have to read?

a. First-order + syntactic reconstruction

\[
\text{[ which-$D \lambda x_e [ \text{have-to [ x is the book that John read ]}]]}
\]

\[
[wh-Q] = \lambda x_e : x \in D.\Box[\lambda w.x = \iota y[book_w(y) \land read_w(y)]]
\]

b. Higher-order ($\Box \gg \pi$) + syntactic reconstruction

\[
\text{[ which-$^H D \lambda \pi_{(et,t)} [ \text{have-to [ $\pi \lambda x_e. x$ is the book that John read ]}]]}
\]

\[
[wh-Q] = \lambda \pi_{(et,t)} : \pi \in ^H D.\Box[\lambda w.\pi(\lambda x_e.x = \iota y[book_w(y) \land read_w(j,y)])]
\]
Appendix. A reconstruction approach

First-order reading

- A copy of *which book* is interpreted within the nucleus. As assumed in categorial approaches, *which book John reads* denotes a one-place predicate.

- **the-insertion** introduces uniqueness, and **variable insertion** introduces a variable bound by the *whP*.

```
CP
  DP
    \lambda x_e C'
      IP
        \Box [\lambda w. x = 1y[book_w(y) \land read_w(j, y)]]
```

- **variable insertion**

```
\lambda y. x = y
```

- **the-insertion**

```
\lambda y[book_w(y) \land read_w(j, y)]
```

```
which-book_w John read_w
```
Appendix. A reconstruction approach

Higher-order reading ($\Box \gg \pi$)

\[
\Box \left[ \lambda w. \pi \left( \lambda x.x = \iota y \left[ \text{book}_w(y) \land \text{read}_w(y) \right] \right) \right]
\]

have to

\[
\lambda w \quad \pi \left( \lambda x.x = \iota y \left[ \text{book}_w(y) \land \text{read}_w(j, y) \right] \right)
\]

variable insertion

\[
\lambda y. \pi \left( \lambda x.x = y \right) \quad \iota y \left[ \text{book}_w(y) \land \text{read}_w(j, y) \right]
\]

THE

the-insertion

\[
\lambda y \left[ \text{book}_w(y) \land \text{read}_w(j, y) \right]
\]

which-book\textsubscript{w} John read\textsubscript{w}
Appendix. A reconstruction approach: Consequences

Conjunctive answers are unacceptable because ‘∧ ⇒ ⊤’ yields a contradiction.

(37) Which book does John have to read?

\[
\llbracket wh-Q \rrbracket = \lambda \pi_{(et,t)} : \pi \in \mathcal{H} \square [\lambda w. \pi (\lambda x. x = 1y [book_w(y) \land read_w(j, y)])]
\]

a. Book A or Book B. 

\[
\llbracket wh-Q \rrbracket (a \uparrow \cup b \uparrow) = \square [\lambda w. [a = 1y [book_w(y) \land read_w(j, y)]] \lor [b = 1y [book_w(y) \land read_w(j, y)]]]
\]

(It has to be the case [that the unique book that John read is Book A or that the unique book that John read is Book B].)

b. #Book A and Book B.

\[
\llbracket wh-Q \rrbracket (a \uparrow \cap b \uparrow) = \square [\lambda w. [a = 1y [book_w(y) \land read_w(j, y)]] \land [b = 1y [book_w(y) \land read_w(j, y)]]]
\]

(#It has to be the case [that the unique book that John read is Book A and that the unique book that John read is Book B].)
Appendix. A reconstruction analysis: Consequences

Recall: Conj-MA arises when $\pi \gg \Diamond$, while Disj-MA is possible in the presence of $\text{Dou}$, regardless of the scope of $\pi$.

**Conj-MA:**
Conjunction over **wide-scope** uniqueness ($\land \gg \iota \gg \Diamond$) yields a contradiction:

(38) Which book can we use [as a textbook] for this class? # Book A and Book B.

$$\llbracket \text{wh-Q} \rrbracket (a \uparrow \cap b \uparrow) = \lambda w.\left[ a = \iota y [\text{book}_w(y) \land \Diamond_w \text{Of}(y)] \right] \land$$
$$\left[ b = \iota y [\text{book}_w(y) \land \Diamond_w \text{Of}(y)] \right]$$

(#$a$ is the unique book that we can use as the only textbook for this class, and $b$ is the unique book that we can use as the only textbook for this class.)
Appendix. A reconstruction analysis: Consequences

Recall: Conj-MA arises when \( \pi \gg \diamond \), while Disj-MA is possible in the presence of \( \text{dou} \), regardless of the scope of \( \pi \).

**Conj-MA:**
Conjunction over **wide-scope** uniqueness \((\land \gg \iota \gg \diamond)\) yields a contradiction:

(38) Which book can we use [as a textbook] for this class?

# Book A and Book B.

\[
\llbracket \text{wh-Q} \rrbracket (a \uparrow \cap b \uparrow) = \lambda w. \left[a = \text{ty}[\text{book}_w(y) \land \diamond_w \text{Of}_w(y)]\right] \land
\left[b = \text{ty}[\text{book}_w(y) \land \diamond_w \text{Of}_w(y)]\right]
\]

(#\(a\) is the unique book that we can use as the only textbook for this class, and \(b\) is the unique book that we can use as the only textbook for this class.)

**Disj-MA:**
Free choice over **narrow-scope** uniqueness \((\text{fc} \gg \diamond \gg \iota)\) does not yield a contradiction:

(39) Which book can we use [as a textbook] for this class?

Book A or Book B.

\[
\llbracket \text{wh-Q} \rrbracket (a \uparrow \cup b \uparrow) = \text{dou} \diamond [\lambda w. (a \uparrow \cup b \uparrow)(\lambda x. x = \text{ty}[\text{book}_w(y) \land \text{Of}_w(y)])]
\]

\[
= [\diamond \lambda w. a = \text{ty}[\text{book}_w(y) \land \text{Of}_w(y)]] \cap
[\diamond \lambda w. b = \text{ty}[\text{book}_w(y) \land \text{Of}_w(y)]]
\]

(\(a\) can be the unique book that we use as the only textbook for this class, and \(b\) can be the unique book that we use as the only textbook for this class.)
References


- Fox, Danny. 2013. Mention-some readings. MIT seminar notes.


- Fox, Danny. 2020. Partition by exhaustification: Towards a solution to Gentile and Schwarz’s puzzle. Manuscript, MIT.


References

