Goals and Facts

Goal: To derive quantifying-into-questions (QoQ) readings uniformly:

Which book did Dqrt-kid read?

Q₁: Which book did every/each kid read?
Q₂: Which book did one/! of the kids read?
Q₃: Which book did none of the kids read?

Pair-list: Which book did x read?

Choice: N.A.

Fact 1: The distribution of Q₁ is preserved with Q₂-coordinations.
   a. Which toy did [very boy and one of the girls] buy?
      ∨ Q₂
   b. Which toy did [very boy and none of the girls] buy?
      Q₂

Fact 2: Q₂ is subject to domain (D-)exhaustivity, but Q₃multi- isn’t.

(2) Context: 200 candidates are competing for 3 job openings.
   a. Guess which candidate will get which job.
   b. Guess which job each candidate will get.

Fact 3: Q₂-embeddings are subject to quantification-(Q)-variability.

(3) For the most part, J knows [Q₂, which book every kid read].
   a. Most x [x is a kid] [J knows which book x read]
   b. Most p [%p is a true ‘kid-x read book-y’ proposition] [J knows p]

Previous studies on Q₂

Function-based approach (Dayal 1996)
Q₂ and Q₃multi- have the same LF and denotation. Each answer is a conjunction naming a function from Dom(V′/v/sub)-obj) to Dom(v/sub-obj).

(4) Q₂/Q₃multi- = (| \{“read(x, f(x)) | kid(x)\} | f \in \{kid \rightarrow book\})

Family-of-questions approach (Fox 2012)
Q₂ and Q₃multi- have different LFs but the same denotation. Q₂ is derived via quantifying-into-predication and moving MIN.

(5) Q₂/Q₃multi- = \{ which book did x read? | kid(x) \}

Framework: Hybrid categorical approach (Xiang 2016, 2018)

- Questions denote functions from short answers to propositional answers.
  - Short answers are extractable from question denotations.
- BeDom turns WhPs from ≥Q₂s into polymorphic domain restrictions.
  - BeDom(v/sub)-hP can combine with any function of a (e-...) type and restrict its domain with Dom(v/sub-hP).

(6) Which book did Ann read?
   a. Individual reading: ‘Which book x is s.t. Ann read x?’
      Q = λx \cdot \langle book(x), read(a, x) \rangle →\langle BeDom(v/h)-bP \rangle λx [Ann read x]
   b. Functional reading: ‘Which function f to book is s.t. Ann f (Ann)?’
      Q = \lambda f : f (f) : \langle book \cdot read (a, f (a)) \rangle →\langle BeDom(v/h)-bP \rangle λf [\langle A read f (f) \rangle]

BeDom(v/sub)-hP binds a functional trace f(x) and restricts its range to Dom(v/sub-hP).

Proposal

The core idea: QoQ readings are functional readings, derived uniformly via quantifying-into-predication (QoQ) and moving the E-minimizer.

(7) Which book did Dqrt-kid read? (Det = ∀/∃/...)
   a. Denotation

\[
\lambda f : \langle book \land \lnot \text{Det}-kidd \ (\text{Dom}(f)) \rangle \land \text{E-min} [K \land \text{Det}-kidd \ (\lambda x . \text{‘read}(f(x)))]
\]

- INPUT: functions mapping Det-kid to atomic books.
- OUTPUT: conjunctions of minimal proposition sets ranging over Det-kid.

In NUCLEUS: (i) QoQ requires K to include Det-sentence in ‘∀x·read(f(x)) | kid(x)’ and f to be defined for Det-kid. (ii) E-ming returns one of the minimal K sets that fulfill these requirements.

Predictions (from nucleus)

- INPUT

| Q₂ | \langle \forall x \cdot \langle \text{read}(f(x)) | \text{kid}(x) \rangle \rangle |
| Q₃ | \langle \forall x \cdot \langle \text{read}(f(x)) | \text{kid}(x) \rangle \rangle |
| Q₄ | \langle \forall x \cdot \langle \text{read}(f(x)) | \text{kid}(x) \rangle \rangle |

- OUTPUT

| D-exh PL ch | + + + |

Quantitative variability of Q₂-embedding

Ans⁵ extracts the complete true short answer (CTSA) of Q₂ — a function. The restriction of the quantificational adverbial are formed by atomic functions that are parts of this CTSA. (cf. Cremers 2018 for composition)

The Q-variability condition

Most \( f'f' \in \text{Arr}(\text{Ans}(Q₂)) \) \( (\text{knowledge}(j, f'f') \leq \text{Ans}(Q₂)(j)) \)

(For most f’s t = ... J knows that f’s is a part of the CTSA of Q₂.)

(8) For the most part, J knows [Q₂, which book did every kid read].
   a. CTSA of Q₂
   b. Restriction of for the most part
   c. The Q-variability condition is true ...

\[ \text{iff } J’s \text{ belief entails the union of the following seven partition cells; iff Most } f'f' \in \text{Arr}(\text{Ans}(Q₂)) \]

Appendix

(9) Answerhood in proposition-based approaches (Dayal and Fox)
   a. \( \text{Ans}(Q₃)(\langle w \rangle) = w \in P \land q \in Q \land q \in Q \land \langle P \leq q \rangle \)
   b. \( \text{Ans}(Q₃)(\langle w \rangle) = \langle \text{Ans}(Q₂)(\langle w \rangle) \rangle \)

(10) Answerhood in categorical approaches
   (Xiang)
   a. \( \text{Ans}(Q₃)(\langle w \rangle) = \alpha \ association \in Q \land q \in \alpha \land q \in Q \land \langle Q \leq \alpha \rangle \)
   b. \( \text{Ans}(Q₃)(\langle w \rangle) = \text{Ans}(Q₂)(\langle w \rangle) \)

(11) Minimizers: minimum vs minimal
   a. \( \text{MIN} = \lambda x . \text{kid}(x) \in Q \land \forall \langle x \rangle \in K \land \langle x \rangle \in K \land \langle x \rangle \)
   b. \( \text{E-min} = \lambda x . \text{kid}(x) \in Q \land \forall \langle x \rangle \in K \land \langle x \rangle \in K \land \langle x \rangle \)
   (Pafeld 1999)

(12) The BeDom-shifter

\( \text{BeDom}(P) = \lambda \theta . P \langle \text{Dom}(P) \land \theta \land \text{Pl}(P) \rangle \land \forall x \in \text{Dom}(P) \land \langle P(a) = \theta(a) \rangle \)


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