Quantifying-into questions

1. Introduction

- Interpretations of questions with quantifiers:

(1) Which book did **every student** read? (Engdahl 1986, 1980)
  a. *Individual reading* ($\iota \gg \forall$)
  'Which book y is s.t. every student read y?' '*Minimalist Program*.'
  b. *Functional reading* ($\iota \gg \forall$)
  'Which function $\mathbf{f}$ to a book is s.t. every student x read $\mathbf{f}(x)$?' 'His favorite book.'
  c. *Pair-list reading* ($\forall \gg \iota$)
  'For every student x, which book did x read?'
  'Andy read *Minimalist Program*, Billy read *Sentence Structure*.'

(2) Which book did **one of the students** read?[1]
  a. *Individual reading* ($\iota \gg \exists$)
  'Which book y is s.t. one of the students student read y?' '*Minimalist Program*.'
  b. *Choice reading* ($\exists \gg \iota$)
  'For one of the students x, which book did x read?'
  'Andy read *Minimalist Program*.'/ 'Billy read *Sentence Structure*.'

Pair-list/choice readings of $\forall/\exists$-questions intuitively involve "quantification into questions (QiQ)," henceforth called **QiQ readings**.

- It is difficult to derive QiQ readings compositionally.

– The LF (3) is ill-formed in Hamblin-Karttunen Semantics: the argument of *every student* shall be of type $\langle e, t\rangle$ (namely, quantifiers shall quantify into truth values), not $\langle e, stt\rangle$.

(3) * [[every student] λx [$_{\text{Q}}$ which book did x read]]

– It's appealing to assume that the quantifier scopes over a covert Q-embedding predicate *ask*/QUEST.

(4) Which book did every students read?
  a. *For every student x, I* ***ask you*** *which book did x read.* (Karttunen 1977)
  b. [[every student]$_{\langle ea,a\rangle}$ λy [QUEST$_{\langle stt,a\rangle}$ [which book did x read]]] (Krifka 2001)

But, the quantification in QiQ is "internal". In embeddings, the quantifier of the embedded question cannot scope over a quantifier in the matrix clause. (Moltmann and Szabolcsi 1994; Fox 2000)

(5) **A waiter** knows which wine **every customer** ordered.
  $\not\approx$ *For every customer x, there is a waiter who know which wine x ordered.* ($\times\forall \gg \exists$)

[1] Functional readings are unavailable in $\exists$-questions: (2) can't be answered by 'his favorite book.'

– No quantification at all? Engdahl (1980): QiQ readings are extensional functional readings.

(6) Which function $f_{\langle e,e\rangle}$ is such that every man x ate $f(x)$?
  a. Every man ate what his wife recommended.
  b. Bill ate the chocolate cake, Tom the rice pudding, and Fred the raspberry pie.

But, QiQ readings and ordinary inverse scope readings are constrained in similar ways; for instance, ∀-quantifiers cannot scope out of declarative finite clauses. (Pafel 1999)

(7) a. A candidate believes that every student voted for him. [× inverse scope]
  b. Which candidate does Susan believe that every student voted for? [× pair-list]

2. Properties of QiQ readings

- **Subject-object/adjunct asymmetry**: QiQ readings are available only when the non-*wh* quantifier serves as the subject and the *wh*-item as an object/adjunct. (Chierchia 1991, 1993)

(8) a. Which candidate did [every student] vote for? √ pair-list
  b. Which student voted for [every candidate]? × pair-list

(9) a. Which candidate did [one of the students] vote for? √ choice
  b. Which student voted for [one of the candidates]? ??choice

Prediction I: The similarity in syntax suggests that QiQ readings shall have similar LFs.

- **Domain exhaustivity**

– Pair-list readings of ∀-questions presuppose domain exhaustivity: every element in the domain of the ∀-subject is paired with an element in the domain of the *wh*-object. (Dayal 2002)

(10) Which book did every student read?
  ⇝ *Every student read a book.*

– In contrast, pair-list readings of multi-*wh* questions are NOT subject to domain exhaustivity. (Xiang 2016, see handout 7);

(11) (Context: 100 candidates are competing for 3 job openings.)
  a. Guess which candidate will get which job.
  b. # Guess which job will every/each candidate get.

Prediction II: The contrast wrt domain exhaustivity suggests that pair-list readings of ∀-questions and multi-*wh* questions are semantically inequivalent and must be derived via different LFs.

- **Distributing pair-list**: only ∀-questions admit pair-list readings.

– Questions with a downward monotone quantifier (Chierchia 1993):

(12) a. 'Who did at most two of the students vote for?'
  # 'Andy voted for Mary, Billy voted for Jenny.'
  b. 'Who did no student vote for?'
  # [silence]

– The seeming pair-list answer in (13) is actually an individual answer with a cumulative reading.

(13) 'Who did two of the students vote for?'
'A and B voted for M and J. In particular, A voted for M, and B voted for J.'

Using a singular *wh*-item removes the confound:

(14) I know that every student voted for a different candidate, please tell me ...
a. Which candidate did every/each student vote for? ($\forall$/EACH $\gg \iota$)
b. # Which candidate did two of the students vote for? ($\exists 2 \gg$ ~~EACH~~ $\gg \iota$)
c. # Which candidate did most of the students vote for? (MOST $\gg$ ~~EACH~~ $\gg \iota$)

Previous analyses either overly predict pair-list (as in Gr&S 1984b, Chierchia 1993), or they rule out such pair-list readings by stipulating machineries that crash in non-$\forall$-questions (as in Gr&S 1984a, Dayal 1996, Pafel 1999, Krifka 2001, Fox 2012b), which comes at the expense of not getting choice readings of $\exists$-questions.

– Coordinating a $\forall$-quantifier with a decreasing quantifier blocks the pair-list reading relative to this $\forall$-quantifier, as in (15a-b); but coordinating it with an $\exists$-quantifier does not, as in (15c).

(15) a. Which toy did [every boy and no girl] buy? ($\times$pair-list for *every boy*)
b. Which toy did [every boy and less than two girls] buy? ($\times$pair-list for *every boy*)
c. Which toy did [every boy and one of the girls] buy? ($\surd$pair-list for *every boy*)

- **Quantificational variability effects** (Berman 1991): in embeddings with a quantificational adverbial (e.g., *mostly*, *for the most part*), $\forall$/multi-*wh* questions with pair-list readings are subject to QV effects:

(16) a. Jenny mostly knows who came.
$\rightsquigarrow$ Most x [x came] [Jenny knows that x came]
b. Jenny mostly knows which paper every/each student read. ($\forall$-question)
$\rightsquigarrow$ Most x [x is a student] [Jenny knows which paper x read]
c. Jenny mostly knows which student read which paper. (multi-*wh* question)
$\rightsquigarrow$ Most x [x is a student $\wedge$ x read a paper] [Jenny knows which paper x read]

- Theories reviewed and to be reviewed (classified based on the treatment of quantification):

  – No quantifying-in (Engdahl)
  – Quantifying into speech acts (Karttunen, Krifka)
  – Quantifying into partitions (Gr&S-a)
  – Quantifying over minimal witness sets (Gr&S-b, Chierchia, Dayal)
  – Quantifying into predication (Fox, Xiang)

  Analyses not covered in this talk: Szabolcsi (1997); Beghelli (1997); Pafel (1999); Preuss (2001); Nicolae (2013); Ciardelli and Roelofsen (2018); and more ...

3. Quantification into partition (Groenendijk & Stokhof 1984a)

- A question denotes a partition of possible worlds (of type $\langle s, st \rangle$). Given an evaluation worlds w, a world w' is in the same partition cell as w iff this question has the same true answers in w and w'.

(17) Who came?
$\lambda w \lambda w' [(\lambda x.\text{came}_w(x)) = (\lambda x.\text{came}_{w'}(x))]$

With only two relevant individuals John and Mary, this partition can be represented as:

w: $\text{came}_w = \{j, m, j \oplus m\}$	=	w: only j and m came in w
w: $\text{came}_w = \{j\}$		w: only j came in w
w: $\text{came}_w = \{m\}$		w: only m came in w
w: $\text{came}_w = \varnothing$		w: nobody came in w

- The $\forall$-question (18) denotes a partition that specifies the student-read-book pair for every student.

(18) Which book did every student read?
$\lambda w \lambda w'.\forall x[\text{student}_@(x) \rightarrow [(\lambda y[\text{book}_@(y).\text{read}_w(x,y)]) = (\lambda y[\text{book}_@(y).\text{read}_{w'}(x,y)\}])]]$
($\lambda w \lambda w'$: for **every student** x is s.t. x read the same book in w and in w'.)

With only two relevant students $s_1 s_2$ and two books $b_1 b_2$, this partition can be represented as:

w: $\text{read}_w = [s_1 \rightarrow b_1, s_2 \rightarrow b_1]$
w: $\text{read}_w = [s_1 \rightarrow b_2, s_2 \rightarrow b_2]$
w: $\text{read}_w = [s_1 \rightarrow b_1, s_2 \rightarrow b_2]$
w: $\text{read}_w = [s_1 \rightarrow b_2, s_2 \rightarrow b_1]$

- **Problems**:

(i) It uses a special quantify-in rule to get quantifying into partitions. (Details omitted)

(ii) this approach cannot extend to $\exists$-questions.

(19) Which book did one of the students read?
$\lambda w \lambda w'.\exists x[\text{student}_w(x) \wedge [(\lambda y[\text{book}_w(y).\text{read}_w(x,y)]) = (\lambda y[\text{book}_{w'}(y).\text{read}_{w'}(x,y)\}])]]$
($\lambda w \lambda w'$: for **one student** x is s.t. x read the same book in w and in w'.)

Consider the four worlds in (i), then (19) denotes the function f in (ii):

(i)
w_1: $[s_1 \rightarrow b_1, s_2 \rightarrow b_1]$
w_2: $[s_1 \rightarrow b_2, s_2 \rightarrow b_2]$
w_3: $[s_1 \rightarrow b_1, s_2 \rightarrow b_2]$
w_4: $[s_1 \rightarrow b_2, s_2 \rightarrow b_1]$

(ii) $$f = \begin{bmatrix} w_1 & \rightarrow & \{w_1, w_3, w_4\} \\ w_2 & \rightarrow & \{w_2, w_3, w_4\} \\ w_3 & \rightarrow & \{w_1, w_2, w_3\} \\ w_4 & \rightarrow & \{w_1, w_2, w_4\} \end{bmatrix}$$

- f isn't a partition; the 'cells' are overlapped (Krifka 2001).
- There is no student x s.t. we can identify which book x read. For example, with the evaluation world w_1, since $f(w_1) = \{w_1, w_3, w_4\}$, we can't tell whether s_1 read b_1 (as in w_1 and w_3) or he read b_2 (as in w_4), nor tell whether s_2 read b_1 or b_2.

4. Quantifying over the minimal witness set

4.1. Groenendijk and Stokhof (1984)

- The QiQ reading involves quantification over the **minimal witness set (MWS)** of the quantifier.

(20) $[\![$Which book did $\mathcal{P}$ read?$]\!]_{\text{QiQ}} = [\![$which member of A read which book$]\!]$ where $\text{MWS}(\mathcal{P}, A)$

(21) Live-on sets and minimal witness sets (Barwise and Cooper 1981)
  a. A generalized quantifier $\mathcal{P}$ lives on a set B iff for any set C, $C \in \mathcal{P} \Leftrightarrow C \cap B \in \mathcal{P}$.
  b. If $\mathcal{P}$ lives on a set B, A is a MWS of $\mathcal{P}$ iff $A \subseteq B$, $A \in \mathcal{P}$, and $\neg\exists A' \subset A[A \in \mathcal{P}]$.

For example, let student $= \{a, b, c\}$, then:

$\mathcal{P}$	MWSs of $\mathcal{P}$	Predicted QiQ reading of 'which book did $\mathcal{P}$ read?'	
every student	$\{a, b, c\}$	[+exh, +pair-list, −choice]	✓
one of the students	$\{a\}, \{b\}, \{c\}$	[−exh, −pair-list, +choice]	✓
two of the students	$\{a, b\}, \{b, c\}, \{a, c\}$	[−exh, +pair-list, +choice]	✗
no student	$\varnothing$	Unavailable	✓
at most two students	$\varnothing$	Unavailable	✓

- **Advantages:** This account explains ...

(i) the domain exhaustivity of ∀-questions:
the MWS of a ∀-quantifier is exhaustive. (Cf., MWSs of ∃-quantifiers are not exhaustive)

(22) Which book did every student read?
$= [\![$which member in A read which book$]\!]$ where $A = \{a, b, c\}$

(ii) choice readings of ∃-questions:
an ∃-quantifier has multiple MWSs, each of which provides a choice.

(23) Which book did one of the students read?
$= [\![$which member in A read which book$]\!]$ where $A \in \{\{a\}, \{b\}, \{c\}\}$

(iii) the unavailability of QiQ readings in questions with a downward monotone quantifier:
the MWS of a downward monotone quantifiers is $\varnothing$.

(24) Which book did no student read?
$= [\![$which member in A read which book$]\!]$ where $A = \varnothing$

Problem: The predicted QiQ reading is pair-list as long as the MWSs of $\mathcal{P}$ are non-singleton. Thus this account overly predicts pair-list readings for non-∀-questions like (25).

(25) Which book did two of the students read?
$= [\![$which member in A read which book$]\!]$ where $A \in \{\{a, b\}, \{b, c\}, \{a, c\}\}$
  a. Andy and Billy read *Minimalist Program*.
  b. # Andy read *Minimalist Program*, Billy read *Sentence Structure*.

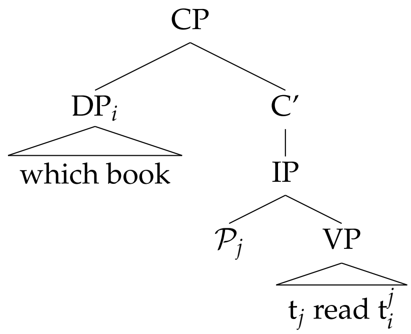
4.2. Chierchia (1993)

- **Motivation**: Both QiQ and functional readings are subject to subject-object asymmetry.[2]

(26) a. Which book did **every student** read? ($\surd$ Individual, $\surd$ Functional, $\surd$ Pair-list)
b. Which student read **every book**? ($\surd$ Individual, $\times$ Functional, $\times$ Pair-list)
i. # Its owner (read every book).
ii. # Andy read B1, Billy read B2.

- **Proposal**: QiQ readings are special functional readings (inspired by Engdahl 1980).

(27) Which book did $\mathcal{P}$ read?
a. Syntax: *wh*-movement leaves a **complex trace** t_i^j. The functional index i is bound by the *wh*-item, and the argument index j is bound by the quantifier.



b. Semantics: (27) denotes a **family of sub-questions**, each of which quantifies over a set A that is a MWS of $\mathcal{P}$ (inspired by Gr&S). A also restricts the domain of the function **f**.

$$\lambda Q.\exists A[\textsc{mws}(\mathcal{P}, A) \wedge Q = \lambda p.\exists x \exists \mathbf{f}[x \in A \wedge \mathbf{f} \in [A \to \mathsf{book}_@] \wedge p = {}^\wedge\mathsf{read}(x, \mathbf{f}(x))]]$$
$$= \Big\{\{{}^\wedge\mathsf{read}(x, \mathbf{f}(x)) \mid x \in A, \mathbf{f} \in [A \to \mathsf{book}_@]\} \mid \textsc{mws}(\mathcal{P}, A)\Big\}$$

(28) a. ⟦Which book did every student read?⟧
$= \{\ \{{}^\wedge\mathsf{read}(s_1, b_1), {}^\wedge\mathsf{read}(s_2, b_2), {}^\wedge\mathsf{read}(s_1, b_2), {}^\wedge\mathsf{read}(s_2, b_2)\}\ \}$
b. ⟦Which book did one of the students read?⟧
$$= \left\{ \begin{array}{l} \{{}^\wedge\mathsf{read}(s_1, b_1), {}^\wedge\mathsf{read}(s_1, b_2)\}, \\ \{{}^\wedge\mathsf{read}(s_2, b_1), {}^\wedge\mathsf{read}(s_2, b_2)\} \end{array} \right\}$$
c. ⟦Which book did no student read?⟧
$= \{\varnothing\}$

- **Advantages & problems:**

(i) It inherits the advantages and problems of Gr&S's witness sets-based account.
(ii) It doesn't explain the domain exhaustivity and pair-list uniqueness effects of the pair-list readings of $\forall$-wh_{SG}-questions.

(29) Which book did every student read?
⇝ *Every student read a book.* Domain exhaustivity
⇝ *None of the students read more than one book.* Pair-list uniqueness

[2] Chierchia subsumes this syntactic asymmetry under the weak crossover constraint. Details omitted.

4.3. Dayal (1996, 2017): function-based Crazy C approach

- **Motivation & Proposal** : In a $\forall$-wh_{SG}- (or multi-wh_{SG}-) question with a pair-list reading, the object-*wh* exhibits **functional dependency** relative to the subject-$\forall/wh$. Thus, Dayal proposes another function-based account which derives pair-list readings of multi-*wh* questions and $\forall$-questions uniformly.

– A question denotes a set of propositions. Each proposition is based on a function from the domain of the $\forall/wh$-subject (viz. $\mathsf{student}_@$) to the domain of the *wh*-object (viz. $\mathsf{book}_@$). In particular, the domain of a $\forall$-quantifier is extracted as retrieving the unique MWS of the quantifier.

(30) Which book did every student read?/ Which student read which book?
$[_Q\ [_{DP}\text{ which book}]_i\ [[_{DP}\text{ which/every student}]_j\ [C^0_{\text{func}}\ [_{IP}\ t_j \text{ read } t_i^j]]]]$

a. $[\![\text{IP}]\!] = \lambda \mathbf{f}_{\langle e,e\rangle}\lambda x_e.\hat{}\mathsf{read}(x, \mathbf{f}(x))$

b. $[\![\text{C}^0{}_{\text{func}}]\!] = \lambda q_{\langle ee,est\rangle}\lambda D\lambda R\lambda p.\exists \mathbf{f} \in [D \to R][p = \bigcap \lambda p'.\exists x \in D[p' = q(x)(\mathbf{f})]]$

c. $Q = \lambda p.\exists \mathbf{f} \in [\mathsf{student}_@ \to \mathsf{book}_@][p = \bigcap \lambda p'.\exists x \in \mathsf{student}_@[p' = \hat{}\mathsf{read}(x, \mathbf{f}(x))]]$
$= \{\bigcap\{\hat{}\mathsf{read}(x, \mathbf{f}(x)) \mid x \in \mathsf{student}_@\} \mid \mathbf{f} \in [\mathsf{student}_@ \to \mathsf{book}_@]\}$

– Applying the ANS-D-operator returns the strongest true member in this proposition set.

Discussion: We reviewed Dayal's Crazy C function-based approach in the context of deriving pair-list readings of multi-*wh* questions (see handout 7). Which of the mentioned advantages and problems extend to the derivation of pair-list readings of $\forall$-questions?

- **Advantages:**

(i) This account keeps the semantic type of questions low, uniformly $\langle st, t\rangle$.

(ii) By virtue of the $\cap$-closure and the ANS-D-operator, this account captures the domain exhaustivity and pair-list uniqueness effects of $\forall$-wh_{SG}-questions (recall (29)).

(31) (w: as for the two relevant students $s_1 s_2$, s_1 read only b_1, **but** s_2 **read both** b_1 **and** b_2.)

a. $Q_w = \{\hat{}\mathsf{read}(s_1, b_1) \cap \hat{}\mathsf{read}(s_2, b_1), \hat{}\mathsf{read}(s_1, b_1) \cap \hat{}\mathsf{read}(s_2, b_2)\}$

b. ANS-D$(Q)(w)$ is undefined

- **Problems:**

– Problems with composition: (i) unconventional meaning of IP and crazy functional C^0; (ii) λ-abstracts are isolated from the moved phrases; (iii) it is syntactically impermissible to move a non-interrogative phrase *every student* to the spec of an interrogative CP (Heim 2012).

– Problems beyond composition:

(i) Composing pair-list readings of $\forall$- and multi-*wh*- questions via the very same LF, this account cannot explain their contrast wrt domain exhaustivity.

(ii) It cannot account for the QV inferences in quantified question embeddings: conjuncts of a conjunctive proposition cannot be retrieved out of this proposition (Lahiri 2002)

(iIi) It over generates $\forall$-pair-list readings for $\exists$-questions. Since $[\![\text{which student}]\!] = [\![\text{some student}]\!]$, it predicts that the $\exists$-question (32a) has a $\forall$-pair-list reading derived through the LF (32b).

(32) a. Which book did some student read?

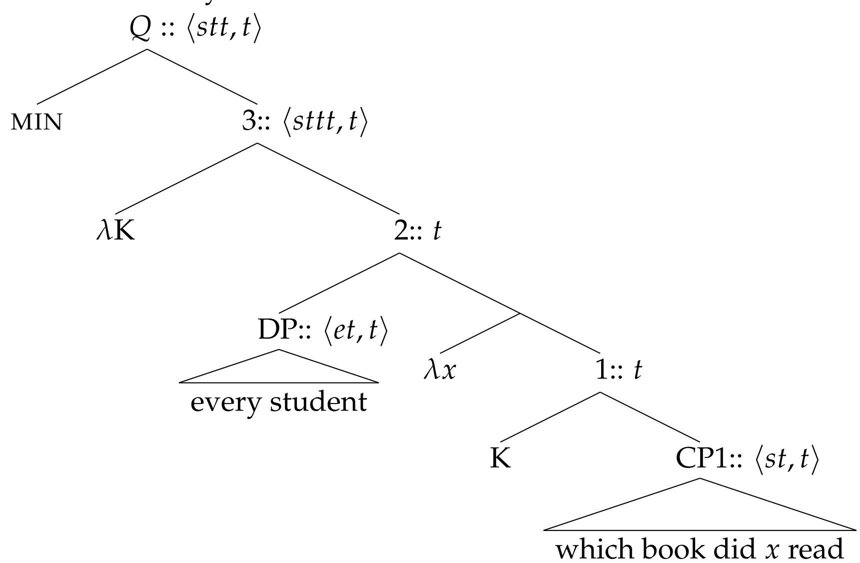
b. $[_{CP}\ [_{DP}\text{ which book}]_i\ [[_{DP}\text{ some student}]_j\ [C^0_{\text{func}}\ [_{IP}\ t_j \text{ read } t_i^j]]]]$

5. Fox (2012): family-of-questions approach

- **Proposal:** A $\forall$-question with a pair-list reading denotes **a family of sub-questions**.

– Composition: (i) the $\forall$-quantifier takes QR and quantifies into a predication (node 1$\sim$2); (ii) abstracting a null K-operator returns a set of sets containing all the sub-questions (node 3); (iii) applying a MIN-operator returns the set of sub-questions.

(33) Which book did every student read?



a. $[\![\text{CP1}]\!] = \{\hat{}\mathsf{read}(x, y) \mid y \in \mathsf{book}_@\}$

b. $[\![1]\!] = \mathrm{K}(\{\hat{}\mathsf{read}(x, y) \mid y \in \mathsf{book}_@\})$

c. $[\![2]\!] = \forall x[x \in \mathsf{student}_@ \rightarrow \mathrm{K}(\{\hat{}\mathsf{read}(x, y) \mid y \in \mathsf{book}_@\})]$

d. $[\![3]\!] = \lambda \mathrm{K}.\forall x[x \in \mathsf{student}_@ \rightarrow \mathrm{K}(\{\hat{}\mathsf{read}(x, y) \mid y \in \mathsf{book}_@\})]$

e. $\textsc{min} = \lambda\alpha.\iota\mathrm{K}[\mathrm{K} \in \alpha \wedge \forall \mathrm{K}' \in \alpha[\mathrm{K} \subseteq \mathrm{K}']]$ (Pafel 1999)

f. $Q = \textsc{min}([\![3]\!]) = \{\{\hat{}\mathsf{read}(x, y) \mid y \in \mathsf{book}_@\} \mid x \in \mathsf{student}_@\}$

– Answerhood: the (complete true) answer to a family of sub-questions is the conjunction of the (complete true) answers to each sub-question.

(34) $$\textsc{Ans}_{\textsc{pw}} = \lambda \mathrm{Q}\lambda w. \begin{cases} \textsc{Ans}\text{-D}(\mathrm{Q})(w) & \text{Q is of type } \langle st, t\rangle \\ \bigcap\{\textsc{Ans}_{\textsc{pw}}(\alpha)(w) \mid \alpha \in \mathrm{Q}\} & \text{otherwise} \end{cases}$$

- **Advantages**
  - It inherits the advantages of Dayal in getting domain exhaustivity and point-wise uniqueness.
  - It manages to treat the $\forall$-quantification as a regular $\forall$-quantification.
  - It leaves room for getting QV inferences of quantified question-embedding. The restriction domain of the quantifier is simply the set of sub-questions contained in the question denotation.

(35) a. Jenny mostly knows [$_{\mathrm{Q}}$ which book every student read].

b. Most Q′ [Q′ $\in$ Q] [Jenny knows Q′]

  - It captures the limited distribution of pair-list readings: the MIN-operator is undefined whenever the quantifier is not $\forall$. For instance, (36a-b) do not have a minimal K set.

(36) a. {K | MOST students x are such that 'which book did x read' is in K}
b. {K | TWO students x are such that 'which book did x read' is in K}

- **Problems:**
  - This account doesn't work for choice readings of ∃-questions: (37) doesn't have a minimal K set.

    (37) {K | ONE student x is such that 'which book did x read' is in K}

  - Fox (2012ab) predicts that pair-list readings of ∀-questions and multi-*wh*-questions are semantically equivalent, which therefore cannot explain their contrast wrt domain exhaustivity.

6. Summary

- Properties of QiQ
  - QiQ readings uniformly exhibit subject-object asymmetry, while pair-list readings of ∀/multi-*wh* questions differ wrt domain exhaustivity.
  - Pair-list readings are only available in ∀-questions. Existing accounts explain this distribution at the cost of not getting choice readings of ∃-questions.
  - In embeddings, ∀/multi-*wh* questions with pair-list readings are subject to QV effects.
- Comparison wrt empirical predictions

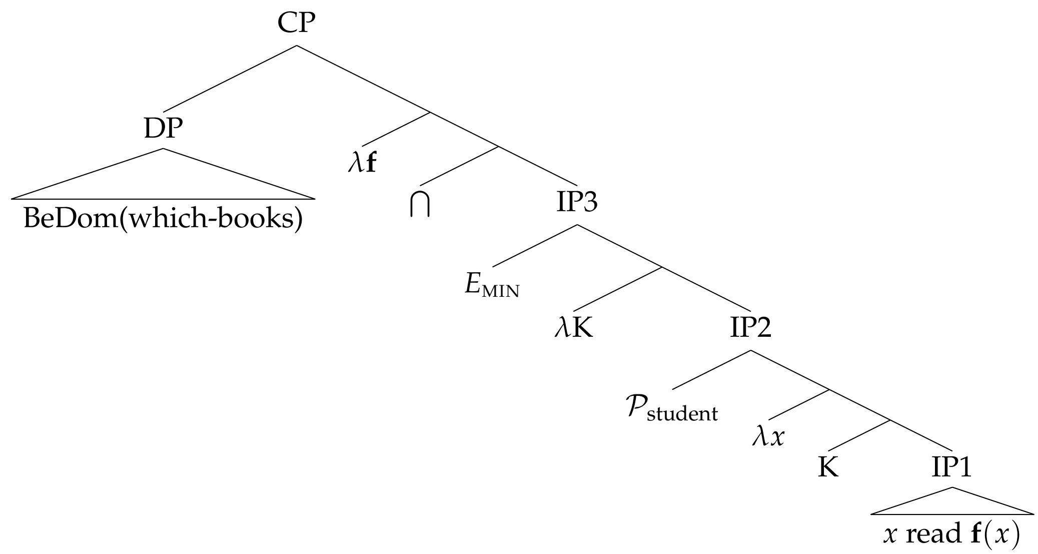
Issues/properties	Gr&S (partition)	Gr&S (mws)	Chierchia	Dayal	Fox
Distributing pair-list	✓	✗	✗	✗	✓
∃-questions	✗	✓	✓	✗	✗
Domain exh of ∀-Q	✓	✓	✗	✓	✓
$[\![\forall\text{-Q}]\!] \neq [\![\text{multi-}wh\text{-Q}]\!]$	✗	✗		✗	✗
QV effects				✗	✓

7. Xiang (2016)

7.1. Basic derivation

- For basics of hybrid categorial approach and deriving functional readings, see handout 3 and 7.
- QiQ readings are uniformly derived as in (38).
  - The question nucleus $\bigcap$[IP3] is structured *a la* Fox (2012), read as "the conjunction of a minimal K s.t. the proposition 'x read $\mathbf{f}(x)$' is in K for $\mathcal{P}_{\text{student}}\ x$."
  - *Wh*-movement leaves a functional trace, whose argument is bound by the subject-quantifier (*pace* Engdahl-Chierchia-Dayal).

(38) Which books did $\mathcal{P}_{\text{STUDENT}}$ read? ($\mathcal{P}_{\text{student}}$: a GQ over student(s).)



- First, in $Q_\forall$, the quantificational predication denoted by IP2 is defined only if the function $\mathbf{f}$ is defined for every student, yielding indefeasible domain exhaustivity.

(39) If $\mathcal{P}_{\text{student}}$ is *every/each student*: $[\![\text{IP2}]\!] = \forall x[x \in \mathsf{student}_@ \rightarrow \text{K}(\mathsf{read}(x, \mathbf{f}(x)))]$

- Second, unlike Pafel-Fox's MIN-operator, the E_{MIN}-operator ($\approx$ Winter's (2001) collectivity raising operator) doesn't presuppose uniqueness. Hence, this LF works for choice readings of $\exists$-questions.

(40) $[\![E_{\text{MIN}}]\!] = \lambda\alpha.f_{\text{CH}}\{\text{K} \mid \text{K} \in \alpha \wedge \forall \text{K}' \in \alpha[\text{K} \not\subset \text{K}']\}$

  - In $Q_\forall$, IP3 denotes a non-singleton set (consists of all the propositions of the form 'student x read book $\mathbf{f}(x)$'), yielding (universal) pair-list.

(41) a. $[\![\text{IP3}]\!] = \mathsf{stdt}_@ \subseteq \text{Dom}(\mathbf{f}).\{\hat{}\mathsf{read}(x, \mathbf{f}(x)) \mid x \in \mathsf{stdt}_@\}$
b. $[\![Q_\forall]\!] = \lambda\mathbf{f}\colon \text{Ran}(\mathbf{f}) \subseteq \mathsf{book}_@ \wedge \mathsf{stdf}_@ \subseteq \text{Dom}(\mathbf{f}).\bigcap\{\hat{}\mathsf{read}(x, \mathbf{f}(x)) \mid x \in \mathsf{stdt}_@\}$

  - In $Q_\exists$, IP3 denotes a singleton set with an unfixed value, giving rise to a choice flavor.

(42) a. $[\![\text{IP3}]\!] = \mathsf{stdt}_@ \cap \text{Dom}(\mathbf{f}) \neq \varnothing.\{\hat{}\mathsf{read}(x, \mathbf{f}(x)) \mid x \in \mathsf{stdt}_@\}$
b. $[\![Q_\exists]\!] = \lambda\mathbf{f}\colon \text{Ran}(\mathbf{f}) \subseteq \mathsf{book}_@ \wedge \mathsf{stdf}_@ \subseteq \text{Dom}(\mathbf{f}).\bigcap\{\hat{}\mathsf{read}(x, \mathbf{f}(x)) \mid x \in \mathsf{stdt}_@\}$

Discussion: If $\mathcal{P}$ is decreasing, what's the denotation of IP3 and the full CP?

7.2. QV inferences

- The same as in a multi-*wh* question, the quantification domain of the matrix quantificational adverbial can be formed by atomic subparts of a short answer (i.e., a Skolem function from students to books).

(43) $\text{Q}_\forall$: Which book did every student read?
(*w: Student $s_1 s_2 s_3$ read book $b_1 b_2 b_3$ in w respectively.*)
a. $[\![\text{Q}_\forall]\!] = \lambda \mathbf{f}\colon \text{Ran}(\mathbf{f}) \subseteq \textsf{book}_@ \wedge \textsf{stdf}_@ \subseteq \text{Dom}(\mathbf{f}). \bigcap\{{}^\wedge\textsf{read}(x, \mathbf{f}(x)) \mid x \in \textsf{stdt}_@\}$
b. $\text{A}\textsc{ns}^S([\![\text{Q}_\forall]\!])(w) = [s_1 \to b_1, s_2 \to b_2, s_3 \to b_3]$
c. $\text{A}\textsc{t}(\text{A}\textsc{ns}^S([\![\text{Q}_\forall]\!])(w)) = \{[s_1 \to b_1], [s_2 \to b_2], [s_3 \to b_3]\}$

– NB: However, the QV inference for an indirect ∀-question cannot be schematized in the same way as what we saw for indirect multi-*wh* question. Can you tell why?

(44) Jenny mostly knows $\text{Q}_\forall$.
$\lambda w.\text{M}\textsc{ost}\ \text{f}'[\text{f}' \in \text{A}\textsc{t}(\text{A}\textsc{ns}^S([\![\text{Q}_\forall]\!])(w))]\ \textsf{know}_w(j, [\![\text{Q}_\forall]\!](\text{f}'))$ (Incorrect)

Alternatively, the scope of QV the inference involves Jenny knowing a "sub-divisive inference".

(45) $\lambda w.\text{M}\textsc{ost}\ \text{f}'[\text{f}' \in \text{A}\textsc{t}(\text{A}\textsc{ns}^S([\![\text{Q}_\forall]\!])(w))][\textsf{know}_w(j, \lambda w'.\text{f}' \leq \text{A}\textsc{ns}^S(P)(w'))]$
(For most f′ that are atomic subparts of the strongest true short answer of $\text{Q}_\forall$, J knows f′ is a subpart of the strongest true short answer of $\text{Q}_\forall$.)

This sub-divisive inference is true iff in every world w' that is compatible with Jenny's belief, the strongest short answer of the embedded $\text{Q}_\forall$ in w' is one of the functions list in the following partition:

	$\begin{bmatrix} s_1 \to b_2 \\ s_2 \to b_2 \\ s_3 \to b_3 \end{bmatrix}$	$\begin{bmatrix} s_1 \to b_3 \\ s_2 \to b_2 \\ s_3 \to b_3 \end{bmatrix}$
$\begin{bmatrix} s_1 \to b_1 \\ s_2 \to b_1 \\ s_3 \to b_3 \end{bmatrix}$	$\begin{bmatrix} s_1 \to b_1 \\ s_2 \to b_2 \\ s_3 \to b_3 \end{bmatrix}$	$\begin{bmatrix} s_1 \to b_1 \\ s_2 \to b_3 \\ s_3 \to b_3 \end{bmatrix}$
$\begin{bmatrix} s_1 \to b_1 \\ s_2 \to b_2 \\ s_3 \to b_1 \end{bmatrix}$	$\begin{bmatrix} s_1 \to b_1 \\ s_2 \to b_2 \\ s_3 \to b_2 \end{bmatrix}$	

(Each cell represents a set of worlds where the student-read-book pairs are as the function enclosed.)

References

Barwise, Jon, and Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4:159–219.

Beghelli, Filippo. 1997. The syntax of distributivity and pair-list readings. In *Ways of scope taking*, 349–408. Springer.

Berman, Stephen Robert. 1991. On the semantics and logical form of wh-clauses. Doctoral Dissertation, University of Massachusetts, Amherst.

Chierchia, Gennaro. 1991. Functional wh and weak crossover. In *West Coast Conference on Formal Linguistics*, volume 10, 75–90.

Chierchia, Gennaro. 1993. Questions with quantifiers. *Natural Language Semantics* 1:181–234.

Ciardelli, Ivano, and Floris Roelofsen. 2018. An inquisitive perspective on modals and quantifiers. *Annual Review of Linguistics* 4.

Dayal, Veneeta. 1996. *Locality in Wh Quantification: Questions and Relative Clauses in Hindi*. Dordrecht: Kluwer.

Dayal, Veneeta. 2002. Single-pair versus multiple-pair answers: Wh-in-situ and scope. *Linguistic Inquiry* 33:512–520.

Dayal, Veneeta. 2017. *Questions*. Oxford: Oxford University Press.

Engdahl, Elisabet Britt. 1980. The syntax and semantics of questions in Swedish. Doctoral Dissertation, University of Massachusetts.

Engdahl, Elizabet. 1986. *Constituent questions*. Dordrecht: Reidel.

Fox, Danny. 2000. *Economy and semantic interpretation*. 35. Cambridge, MA: MIT Press.

Fox, Danny. 2012. Pair-list with universal quantifiers. MIT class notes.

Groenendijk, Jeroen Antonius Gerardus, and Martin Johan Bastiaan Stokhof. 1984. Studies on the semantics of questions and the pragmatics of answers. Doctoral Dissertation, Univ. Amsterdam.

Karttunen, Lauri. 1977. Syntax and semantics of questions. *Linguistics and philosophy* 1:3–44.

Krifka, Manfred. 2001. Quantifying into question acts. *Natural Language Semantics* 9:1–40.

Moltmann, Friederike, and Anna Szabolcsi. 1994. Scope interactions with pair-list quantifiers. In *Proceedings of NELS*, volume 24, 381–395.

Nicolae, Andreea Cristina. 2013. Any questions? polarity as a window into the structure of questions. Doctoral Dissertation, Harvard University.

Pafel, Jürgen. 1999. Interrogative quantifiers within scope. *Linguistics and Philosophy* 22:255–310.

Preuss, Susanne Marie-Luise. 2001. Issues in the semantics of questions with quantifiers. Doctoral Dissertation, Rutgers.

Szabolcsi, Anna. 1997. Quantifiers in pair-list readings. In *Ways of scope taking*, ed. Anna Szabolcsi. Kluwer, Dordrecht.

Xiang, Yimei. 2016. Interpreting questions with non-exhaustive answers. Doctoral Dissertation, Harvard University Cambridge, Massachusetts.