Multi-*wh* questions

1. Introduction

- Multi-*wh* questions are ambiguous between single-pair (SP) readings and pair-list (PL) readings:

(1) Which boy invited which girl?
  a. Andy invited Mary.
  b. Andy invited Mary, Billy invited Jenny, Clark invited Susi.

- Two lines of approaches to derive pair-list readings:
  – **Function-based approaches** (Engdahl 1986, 1980; Dayal 1996, 2017)
  The pair-list reading of a multi-*wh* question inquires the *functional dependency* between the sets that the *wh*-items quantify over.
  – **Family-of-questions approaches** (Hagstrom 1998, Fox 2012a, Nicolae 2013, Kotek 2014)
  Multi-*wh* questions with pair-list readings denote families of questions .

2. The phenomenon: Domain exhaustivity?

- The current dominant view due to Dayal: pair-list readings of questions with multi-*wh* or a universal quantifier are subject to **domain exhaustivity**.

(2) **Presuppositions of a multiple wh_{sg}-question** (Dayal 2002)
  a. *Domain exhaustivity*
  Every member of the set quantified over by the overtly moved *wh*-item is paired with a member of the set quantified over by the in-situ *wh*-item.
  b. *Point-wise uniqueness*
  Every member of the set quantified over by the overtly moved *wh*-item is paired with no more than one member of the set quantified over by the in-situ *wh*-item.

(3) Which boy invited which girl?
  a. Domain exhaustivity: every boy is such that he invited a girl
  b. Point-wise uniqueness: every boy is such that he invited exactly one girl

– Remove the domain restriction confound:
In (3), it is possible that *which boy* quantifies over a subset of boys who invited some girl(s). To avoid this confound, Fox (2012a) adds an explicit domain restriction to each of the *wh*-phrases:

(4) a. Guess which one of these 3 kids will sit on which of these 4 chairs. (^{OK}SP, ^{OK}PL)
  b. Guess which one of these 4 kids will sit on which of these 3 chairs. (^{OK}SP, #PL)

In a pair-list reading, the presupposition of (4b) yields a infelicitous inference that there will be multiple kids sitting on the same chair.

- My view: pair-list readings of multi-*wh* questions are NOT subject to domain exhaustivity.
  - Musical Chairs (contra (4)):

    (5) (Context: *Four kids are playing the game of Musical Chairs and are competing for three chairs.*)
    "Guess which one of these 4 kids will sit on which of these 3 chairs."
    $\not\rightsquigarrow$ *Each of the four kids will sit on one of the three chairs.*

  - Job candidates:
    The multi-*wh* question is not subject to domain exhaustivity, or at least that its domain exhaustivity effect, if any, is much less robust than that of the corresponding ∀-question.

    (6) (Context: *100 candidates are competing for three jobs.*)
    a. √ "Guess which candidate will get which job."
    b. # "Guess which job will every candidate get."

  - Could one say that the domain exhaustivity effect of a multi-*wh* question can be associated with any of the *wh*-phrases? No, consider:

    (7) (Context: *There are four boys and four girls in the dancing class. Each boy will be paired with one girl to participate in a dance competition. Only two pairs will be in the finals.*)
    "Guess which one of the 4 boys will dance with which one of the 4 girls in the finals."
    a. $\not\rightsquigarrow$ *Each boy will dance with some girl in the finals.*
    b. $\not\rightsquigarrow$ *Each girl will dance with some boy in the finals.*

3. Function-based approaches

- Claiming that the pair-list readings of (8a-b) are identical, Engdahl (1980, 1986) and Dayal (1996, 2017) derive them based on similar LFs.

  (8) a. Which girl did every/each boy invite?
  b. Which boy invited which girl?

  This section discusses only Dayal's analysis, because it is so far the only function-based approach that can predict the point-wise uniqueness effects of multi-wh_{SG} questions.

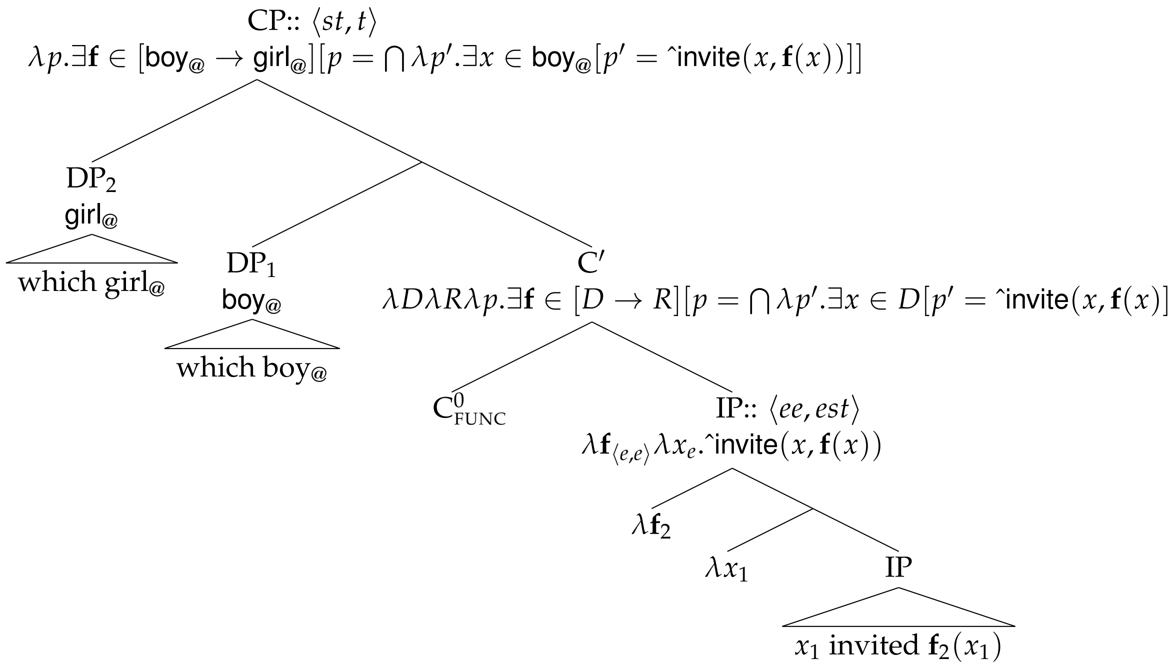
- Dayal argues that pair-list answers to multi-*wh* questions involve a **functional dependency** between the quantification domains of the *wh*-items.

  (9) Which boy invited which girl?
  ≈ 'For which function **f** from boy to girl is s.t. x invited $\mathbf{f}(x)$?'

  This functional dependency is ascribed to a functional C head.

  (10) $[\![\text{C}^0{}_{\text{FUNC}}]\!] = \lambda q_{\langle ee,est\rangle}\lambda D\lambda R\lambda p.\exists \mathbf{f} \in [D \to R][p = \bigcap \lambda p'.\exists x \in D[p' = q(x)(\mathbf{f})]]$
  where $\mathbf{f} \in [D \to R]$ iff $\text{Dom}(\mathbf{f}) = D$ and $\forall x[\mathbf{f}(x) \in R]$
  a. an ∃-closure over functions ($\exists \mathbf{f}$),
  b. restrictions on the domain and range of the function ($\mathbf{f} \in [D \to R]$),
  c. the creation of the graph of each such function. The graph of a function **f** is the conjunction that coordinates all the propositions obtained by quantifying over the domain of **f** (viz., $\bigcap \lambda p'.\exists x \in D[p' = q(x)(\mathbf{f})]$).

(11) Which boy invited which girl?



- Domain restrictions of the *wh*-items saturate the domain (D) and range (R) arguments of $\mathrm{C}^0_{\text{FUNC}}$. The restriction of a *wh*-item can be extract via the BE-shifter, if a *wh*-item is defined as an existential GQ. Alternatively, one can treat the root denotation of a *wh*-item as a set of entities and derive its quantificational meaning via employing an ∃-shifter (Bittner 1994).

(12) a. $\text{BE} = \lambda \mathcal{P}.\lambda z[\mathcal{P}(\lambda y.y = z)]$
b. $\exists = \lambda D \lambda f.\exists x \in D[f(x)]$

- Each proposition in the Hamblin set names a function **f** that is defined for every member in the set quantified by the subject-*wh*, yielding domain exhaustivity.

(13) (With two relevant boys *ab*, and two relevant girls *mj*.)

$$Q = \lambda p.\exists \mathbf{f} \in [\mathsf{boy}_@ \to \mathsf{girl}_@][p = \bigcap \lambda p'.\exists x \in \mathsf{boy}_@[p' = {}^\wedge\mathsf{invite}(x, \mathbf{f}(x))]]$$
$$= \{\bigcap\{{}^\wedge\mathsf{invite}(x, \mathbf{f}(x)) \mid x \in \mathsf{boy}_@\} : \mathbf{f} \in [\mathsf{boy}_@ \to \mathsf{girl}_@]\}$$
$$= \left\{ \begin{array}{ll} {}^\wedge\mathsf{invite}(a,m) \cap {}^\wedge\mathsf{invite}(b,j), & {}^\wedge\mathsf{invite}(a,j) \cap {}^\wedge\mathsf{invite}(b,j) \\ {}^\wedge\mathsf{invite}(a,m) \cap {}^\wedge\mathsf{invite}(b,m), & {}^\wedge\mathsf{invite}(a,j) \cap {}^\wedge\mathsf{invite}(b,m) \end{array} \right\}$$

Applying the ANS-operator returns the unique strongest true proposition in Q and presupposes its existence, yielding point-wise uniqueness. Can you see why?

(14) $\text{ANS-D}(Q)(w) = \exists p[w \in p \in Q \wedge \forall q[w \in q \in Q \to p \subseteq q]].$
$\iota p[w \in p \in Q \wedge \forall q[w \in q \in Q \to p \subseteq q]]$

- **Advantage**: This approach keeps the question type low. Thus, it leaves more space for complex *wh*-constructions that might need a more complex type (e.g., *wh*-triangles, multi-*wh* echo questions).

- **Problems in composition**

  – The semantic type of IP is abnormal; canonically, IP is assumed to be of type t or $\langle s, t\rangle$.
  – C^0_{FUNC} is structure-specific and has a lot of novel semantic features.
  – The λ-operators are kept within IP and are isolated from the moved *wh*-phrases. Moreover, since there are two moved pieces for a single λ-abstract, the binding relations between the moved *wh*-phrases and the *wh*-traces are ambiguous.

- **Problems beyond composition**

  – The $\bigcap$-closure has unwelcome consequences in predicting the quantification variability (QV) effects in question embeddings (Lahiri 2002). The QV involves quantification over propositions of the form 'boy x invited girl y'. Once those propositions are mashed by conjunctions, there is no way to retrieve them back.[1]

    (15) John mostly knows which boy invited which girl.
    $\rightsquigarrow$ Most p [p is a true proposition of the form 'boy x invited girl y'] [John knows p]

  – Pair-list readings of multi-*wh*- and $\forall$- questions are derived via the very same LF; therefore, this account cannot explain their contrast wrt domain exhaustivity.

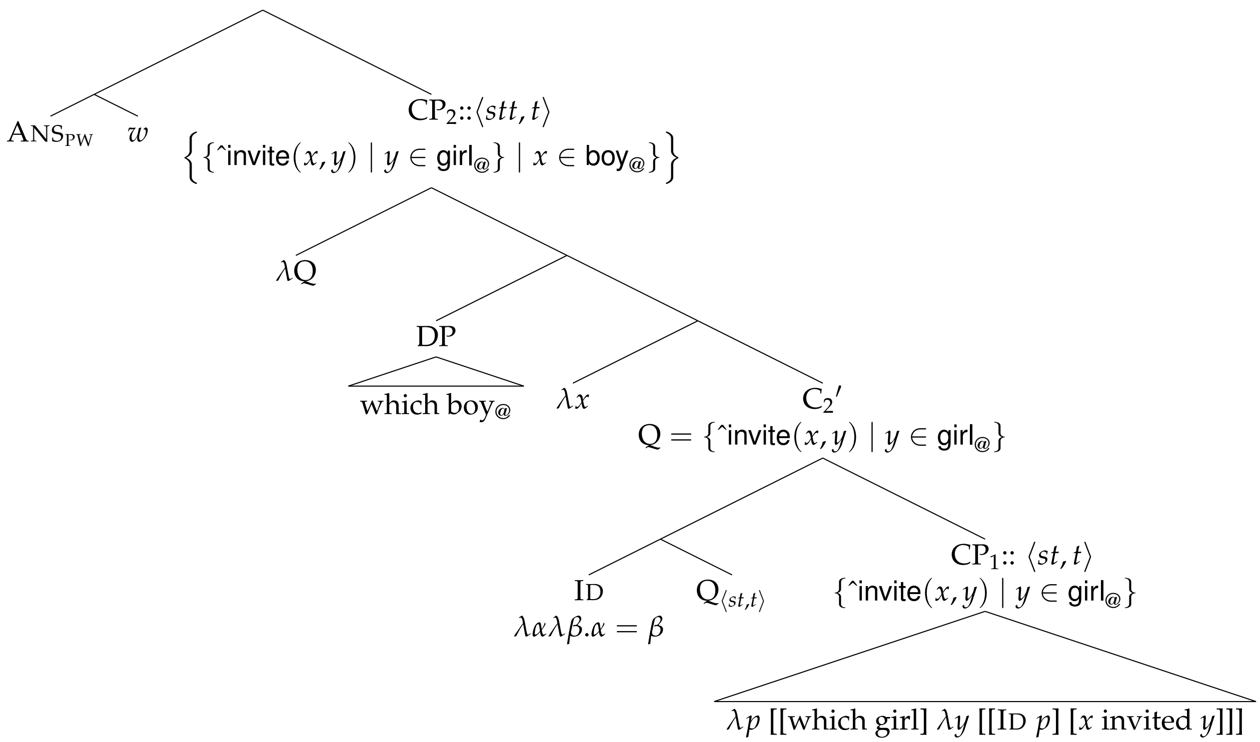
[1] Thus, in a recent colloquium talk at MIT, Dayal (2016) proposes to get rid of the $\bigcap$-closure in C^0_{FUNC} and analyze the root denotation of a multi-*wh* question as a family of proposition sets. This revision manages to keep the atomic propositions alive, but sacrifices the advantage of keeping the semantic type of questions low.

4. Family-of-questions approaches (Fox 2012a)

- Multiple-*wh* questions with pair-list readings denote *families of questions*. Answering a family of questions amounts to answering all of the questions in this family.

(16) $[\![\text{Which boy invited which girl?}]\!] = \big\{[\![\text{which girl did } x \text{ invite?}]\!] \mid x \in \mathsf{boy}_@\big\}$

- Fox's composition follows standard GB-transformed Karttunen Semantics. The key insight is to allow iteration of a type flexible version of the ID-function.



The answerhood-operator applies point-wise and is defined recursively. The point-wise uniqueness effect is captured by point-wise applying Dayal's presuppositional ANS-D-operator. The domain exhaustivity effect is achieved by the application of a $\bigcap$-closure.

(17) $$\text{ANS}_{\text{PW}} = \lambda Q \lambda w. \begin{cases} \text{ANS-D}(Q)(w) & \text{if } Q \text{ is of type } \langle st, t\rangle \\ \bigcap\{\text{ANS}_{\text{PW}}(\alpha)(w) \mid \alpha \in Q\} & \text{otherwise} \end{cases}$$ (Fox 2012a)

- **Advantages**: (i) The composition is hassle-free. (ii) It easily derives the QV inference of a quantified question-embedding — mostly knowing a family of questions amounts to knowing most of the questions in this family.

(18) John mostly knows which boy invited which girl.
Most Q [Q $\in \{[\![x \text{ invited which girl}]\!] \mid x \in \mathsf{boy}_@\}$] [John knows Q]

- **Problems**: Domain exhaustivity comes from the point-wise ANS-operator. While assuming different LFs, Fox (2012a,b) predicts that a multi-*wh* question with a pair-list reading is semantically identical to the corresponding $\forall$-question. Thus, Fox can't explain the contrast wrt domain exhaustivity.

5. A hassle-free function-based approach

- The derivation in this section follows the hybrid categorial approach (Xiang 2016):
  - The root denotation of a question is a topical property, namely, a function from a short answers (a meaning in the *wh*-domain) to the corresponding propositional answer.
  - A *wh*-phrase is an existential generalized quantifier with a possibly polymorphic *wh*-domain. It is further converted into a type-flexible domain restrictor via a BEDOM-operator.

(19) $[\![\text{wh-}]\!] = \lambda A \lambda B.\exists x \in A[B(x)]$
a. $[\![\text{which girl}_@]\!] = [\![\text{which}]\!](\dagger[\![\text{girl}]\!]) = \lambda B.\exists x \in \mathsf{girl}_@[B(x)]$
b. $[\![\text{which girls}_@]\!] = [\![\text{which}]\!](\dagger[\![\text{girls}]\!]) = \lambda B.\exists x \in \dagger{}^*\mathsf{girl}_@[B(x)]$

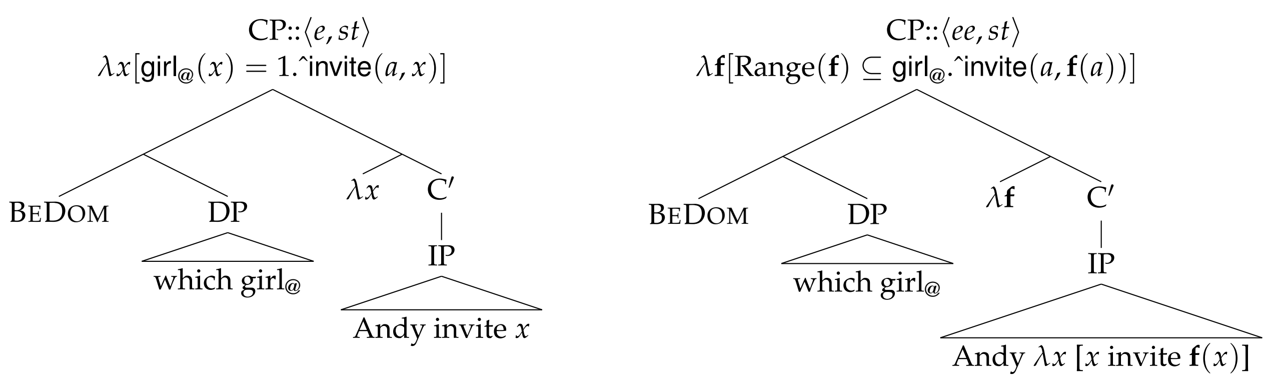
5.1. Introducing functions

- Add Skolem functions into the *wh*-domain.

(20) $[\![\text{wh-}]\!] = \lambda A \lambda B.\exists x \in (A \cup \{\mathbf{f} \mid \text{Range}(\mathbf{f}) \subseteq A\})[B(x)]$

- Basic functional readings
  - If *wh*-movement leaves only a trace of type e, the derived topical property is a property of individuals, yielding an individual reading.
  - If *wh*-movement leaves a functional trace, the obtained topical property is a property of Skolem functions, yielding a functional reading.

(21) Which girl did Andy invite?
a. *Individual reading:*
'Which girl x is s.t. Andy invited x?' 'Mary.'
b. *Functional reading:*
'Which function $\mathbf{f}$ to $\mathsf{girl}_@$ is s.t. Andy invited $\mathbf{f}$(Andy)?' 'His sister.'



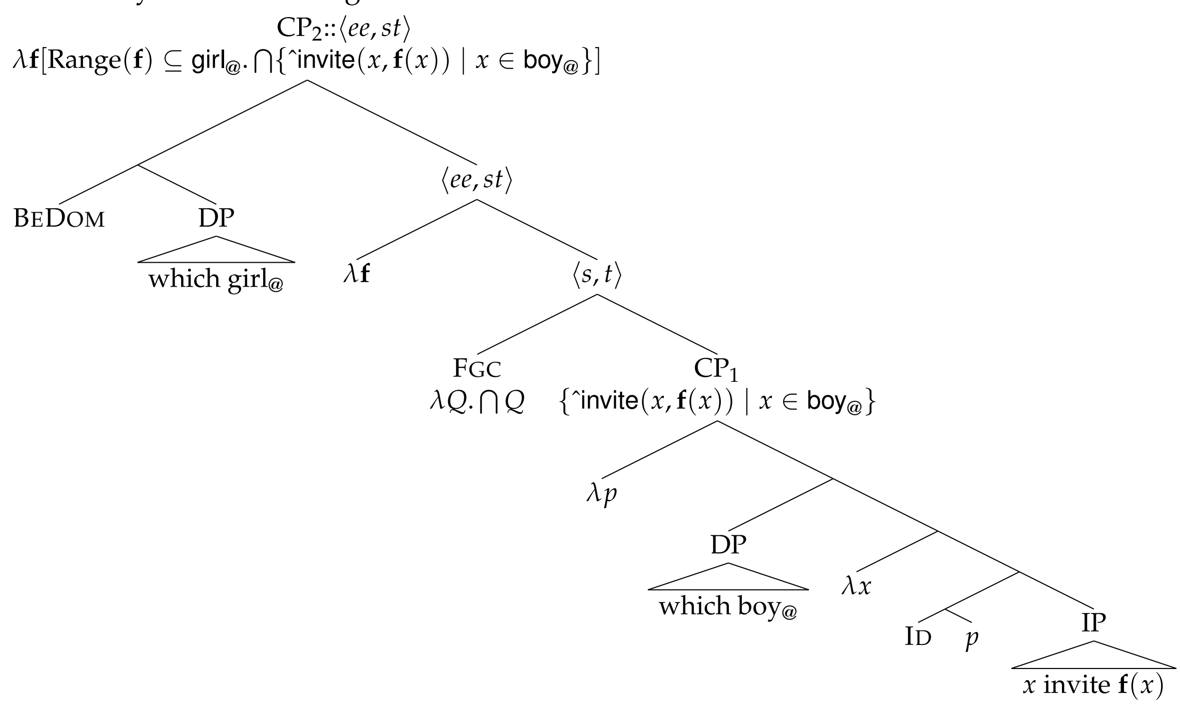
Exercise: Under one reading, an elided disjunction of two functions can completely address the □-question. How can we derive this reading?

(22) 'Who does John have to invite?' 'His dancing partner or his sister.'

5.2. Deriving pair-list readings of multi-*wh* questions

- The LF involves two layers of interrogative CPs.
  - CP_1 denotes a set of propositions, composed the same as in GB-transformed Karttunen Semantics. Next, a $\bigcap$-closure conjoins this set. The $\bigcap$-closure can be viewed as Dayal's *function graph creator* (FGC).
  - Moving 'BEDOM(which girl)' covertly to [Spec, CP_2] leaves a functional trace within IP. The yielded topical property takes a Skolem function ranging over atomic girls and maps this Skolem function to a conjunctive proposition that spells out the graph of this function.

(23) Which boy invited which girl?



- Consequences on domain exhaustivity and uniqueness:
  - The topical property restricts the range of **f**, but not the domain of **f**. Hence, for a function being used as a possible short answer of (23), its domain could be any set overlapped with the restriction of the subject-*wh* (viz., $\mathsf{boy}_@$). Thus, no domain exhaustivity arises.
  - A function **f** maps each element in its domain to exactly one atomic girl. Hence, there won't be answers like $^\wedge\mathsf{invite}(a, j) \cap {}^\wedge\mathsf{invite}(a, m)$. Dayal's presupposition yields point-wise uniqueness.

(24) (Consider two relevant boys *ab* and two relevant girls *mj*)

a. Possible answers are based on the following functions:

$[a \to m]$ $[b \to m]$ $[a \to m, b \to m]$ $[a \to j, b \to m]$
$[a \to j]$ $[b \to j]$ $[a \to m, b \to j]$ $[a \to j, b \to j]$

b. The minimal range of the topical property is the Hamblin set:

$$Q = \left\{ \bigcap\{^\wedge\mathsf{invite}(x, \mathbf{f}(x)) \mid x \in \mathsf{boy}_@\} \mid \text{Range}(\mathbf{f}) \subseteq \mathsf{girl}_@ \right\}$$
$$= \left\{ \begin{array}{llll} ^\wedge\mathsf{invite}(a, m) & ^\wedge\mathsf{invite}(b, m) & ^\wedge\mathsf{invite}(a, m) \cap {}^\wedge\mathsf{invite}(b, m) & ^\wedge\mathsf{invite}(a, j) \cap {}^\wedge\mathsf{invite}(b, m) \\ ^\wedge\mathsf{invite}(a, j) & ^\wedge\mathsf{invite}(b, j) & ^\wedge\mathsf{invite}(a, m) \cap {}^\wedge\mathsf{invite}(b, j) & ^\wedge\mathsf{invite}(a, j) \cap {}^\wedge\mathsf{invite}(b, j) \end{array} \right\}$$

Exercise: Compose the following multi-*wh* question.

(25) Which student read which two or three books?

5.3. Deriving QV inferences

- The quantification domain of *mostly* can be retrieved based on the "short answers". Given a function **f** such that **f** is max-informative true short answer of the embedded question, the quantification domain of matrix quantificational adverb *mostly* is the set of functions that are atomic subsets of **f**.

(26) **Atomic functions**

a. A function **f** is atomic iff $\bigoplus \text{Dom}(\mathbf{f})$ is atomic. For example:

i. Atomic functions: $\mathbf{f}_1 = [a \rightarrow m]$, $\mathbf{f}_2 = [b \rightarrow j]$, $\mathbf{f}_3 = [a \rightarrow m \oplus j]$

ii. Non-atomic functions: $\mathbf{f}_4 = [a \rightarrow m, b \rightarrow j]$, $\mathbf{f}_5 = [a \oplus b \rightarrow j]$

b. $\textsc{At}(\mathbf{f}) = \{\mathrm{f}' \mid \mathrm{f}' \subseteq \mathbf{f} \text{ and } \bigoplus \text{Dom}(\mathrm{f}') \text{ is atomic}\}$

- For example[2]

(27) John mostly knows [$_{\text{Q}}$ which boy invited which girl].
(*w*: *Boys abc each invited only girls jms, respectively. No other boy invited any of the girls.*)

a. The max-inf true short answer of Q:

$$\textsc{Ans}^s([\![\text{Q}]\!])(w) = \begin{bmatrix} a \rightarrow m \\ b \rightarrow j \\ c \rightarrow s \end{bmatrix}$$

b. Atomic subsets of this max-inf true short answer

$$\textsc{At}(\textsc{Ans}^s([\![\text{Q}]\!])(w)) = \left\{ \begin{array}{c} [a \rightarrow m] \\ [b \rightarrow j] \\ [c \rightarrow s] \end{array} \right\}$$

c. The QV inference

$$\textsc{Most}\ \mathrm{f}' \Big[\mathrm{f}' \in \textsc{At}(\textsc{Ans}^s([\![\text{Q}]\!])(w))\Big] \Big[\mathsf{know}(j, \bigcap\{\hat{}\mathsf{invite}(x, \mathrm{f}'(x)) \mid x \in \mathsf{boy}_@\})\Big]$$

$$= \textsc{Most}\ \mathrm{f}' \left[\mathrm{f}' \in \left\{ \begin{array}{c} [a \rightarrow m] \\ [b \rightarrow j] \\ [c \rightarrow s] \end{array} \right\}\right] \Big[\mathsf{know}(j, \bigcap\{\hat{}\mathsf{invite}(x, \mathrm{f}'(x)) \mid x \in \mathsf{boy}_@\})\Big]$$

$$= \textsc{Most}\ \mathrm{f}' \left[\mathrm{f}' \in \left\{ \begin{array}{c} [a \rightarrow m] \\ [b \rightarrow j] \\ [c \rightarrow s] \end{array} \right\}\right] \Big[\mathsf{know}(j, \hat{}\mathsf{invite}(x, \mathrm{f}'(x)))\Big]$$

[2] This example ignores technical complications needed for getting mention-some readings and the QV effects in pair-list readings of indirect ∀-questions.

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