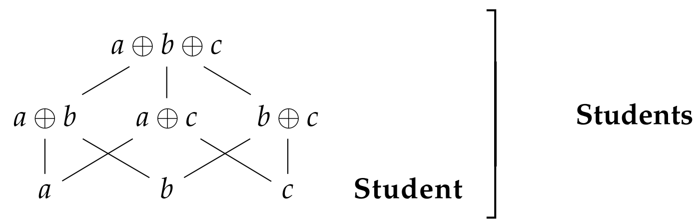
Yimei Xiang, Rutgers University February 26, 2019

Maximality, uniqueness, and negative islands

1. Maximality and uniqueness of *the*-phrases

1.1. Singular/plural

- The ontology of individuals by Sharvy (1980) and Link (1983): a singular term denotes a set of atomic elements, while a plural term denotes a set consisting of both atomic and sum elements.



Treating plurals as sets ranging over not only sums but also atomic elements is called the *weak theory* of plurality (Sauerland et al. 2005, a.o.), in contrast to the *strong theory* that defines plurals as denoting sets consisting of only non-atomic elements. Whether plurals are treated weak or strong is not crucial for this class.

- Formally, the extension of a plural term is obtained by applying a **star (*)-operator** to the extension of the corresponding singular term. This *-operator closes a set of entities under merelogical sum.

(1) $^*A = \{x \mid \exists A' \subseteq A[x = \bigoplus A']\}$ (Link 1983)
(*A is the set that contains any sum of things taken from A.)

1.2. Semantics of *the*-phrases

- *The*-NP_{SG} presupposes uniqueness:[1]

(2) a. [Pointing to one student,] I know the {student, # students}.
b. [Pointing to two students,] I know the {#student, students}.

- Attempt 1: Uniqueness as the unique existence (*the* = ι)

The determiner *the* picks out a contextually relevant entity (i.e., this entity is in the discourse domain D) from the extension of its NP-complement, defined only if such an entity is unique.

(3) $[\![\text{the}_D]\!] = \lambda P_{\langle e,t\rangle} : \exists! x \in D[P(x)].\ \iota x \in D[P(x)]$

This presupposition predicts the uniqueness requirement of *the*-NP_{SG} but is too strong for *the*-NP_{PL} (regardless of defining plurality weak or strong).

(4) [Pointing to two/three students,] I know the students.

[1] In the weak theory of plurality, the anti-uniqueness inference (i.e., 'more than one') of a plural term an **anti-presupposition**, namely, an implicature that the singular correspondent is undefined.

- Attempt 2: Uniqueness as requiring the existence of a maximal element (*the* = MAX)

  '*The*-NP' denotes the maximal element in the extension of the NP-complement, and presupposes the existence of this element.

  (5) $[\![\text{the}_D]\!] = \lambda P_{\langle e,t\rangle} : \exists x \in D[P(x) = 1 \wedge \forall y \in D[P(y) = 1 \rightarrow y \leq x]].$
  $\iota x \in D[P(x) = 1 \wedge \forall y \in D[P(y) = 1 \rightarrow y \leq x]]$

  With two students *ab* in the discourse domain:

  – $[\![\text{student}]\!] = \{a, b\}$ ... doesn't have a maximal member;
  – $[\![\text{students}]\!] = \{a, b, a \oplus b\}$ ... has a maximal element $a \oplus b$.

  Discussion: Following attempt 2, how can we capture the infelicity of the following sentence?

  (6) [Pointing to three students] "# I know the two students."

2. Uniqueness and maximality in *wh*-questions

- The puzzle: a single-*wh* question in which the *wh*-complement is singular-marked can have exactly one true answer.

  (7) a. '**Which student** came? # I heard that many students did.'
  b. '**Which students** came? I heard that many students did.'
  c. '**Who** (among the students) came? I heard that many students did.'

2.1. Encoding uniqueness/maximality into the restriction

- Dayal (1991) builds the uniqueness/maximality presupposition into Karttunen sets by encoding definiteness into the *wh*-**lexicon**:

  (8) Which student came?
  $\lambda p.\exists x[x = \text{MAX}(\lambda y.\mathsf{student}(y) \wedge \mathsf{came}(y)) \wedge p(w) = 1 \wedge p = \hat{}\mathsf{came}(x)]$

  Quantification is over those individuals that maximally satisfy the NP-complement and the question nucleus. (Compare: for *the*-NP, we look for individuals that maximally satisfy the NP-complement)

– Problems:

  1. The question nucleus '$\mathsf{came}(x)$' contributes its meaning in two places.
  2. It is too strong for multi-*wh* questions with pair-list readings.

2.2. Encoding uniqueness/maximality into the nucleus

- Rullmann (1995: section 3.4) encodes maximality into the **question nucleus**:

(9) What did John read?
$\lambda p.\exists x[p(w) = 1 \wedge p = \lambda w[x = \textsc{max}(\lambda y.\mathsf{read}_w(j, y)]]$

- Rullmann and Beck (1998) assume that possible answers *which*-questions are **partial propositions**:

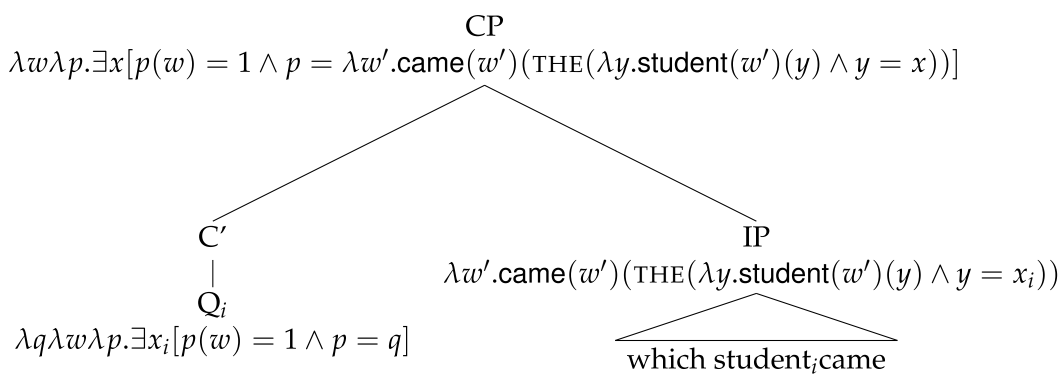
(10) Which student came?
$\{$the student Andy came, the student Billy came, the student Cindy came, ...$\}$

Partial proposition are undefined in certain worlds.

(11) a. $[\![$the student Andy$]\!] = \textsc{the}(\lambda y.\mathsf{student}(w)(y) \wedge y = a))]$
b. $\lambda w.\mathsf{came}(w)(\textsc{the}(\lambda y.\mathsf{student}(w)(y) \wedge y = a))]$ denotes the proposition p such that:
i. $p(w) = 1$ if Andy is an atomic student in w and Andy came in w;
ii. $p(w) = 0$ if Andy is an atomic student in w and Andy didn't come in w;
iii. p is undefined otherwise

Which-phrases are **definites** (of type e). They are interpreted in base position and thus are part of the nucleus. The free variable introduced by the *which*-phrase is bound by the question operator in Comp (*a la* Berman 1991).

(12) a. $[\![$which student$_i]\!] = \textsc{the}(\lambda y.\mathsf{student}(w)(y) \wedge y = x_i))]$
b. Which student came?



c. Consider two relevant students ab. Assume that only a came in w_1, only b came in w_2, both came in w_3, and neither came in w_4, we have:

$$[\![\text{Which student came?}]\!] = \begin{bmatrix} w_1 & \rightarrow & \{\hat{}\mathsf{came}(a)\} \\ w_2 & \rightarrow & \{\hat{}\mathsf{came}(b)\} \\ w_3 & \rightarrow & \\ w_4 & \rightarrow & \varnothing \end{bmatrix}$$

Discussion: Does the above singular-marked question maps w_3 to $\varnothing$ or 'undefined' or ...?

2.3. Encoding maximality into Answerhood

2.3.1 Answerhood

- Heim (1994) defines the answerhood (Ans)-operator for weakly exhaustive answers as a simple conjunction over a Karttunen set.

(13) $\textsc{Ans}\text{-H}(Q)(w) = \bigcap\{p \mid w \in p \in Q\}$
(The conjunction of the propositions in Q that are true in w.)

- Dayal (1996) encodes maximality into the Ans-operator: $\textsc{Ans}\text{-D}(Q)(w)$ returns the strongest true answer and presupposes the existence of this strongest true answer. The presupposition of ANS-D is usually called *Dayal's presupposition*.

(14) $\textsc{Ans}\text{-D}(Q)(w) = \exists p[w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow p \subseteq q]].$
$\iota p[w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow p \subseteq q]]$
($\textsc{Ans}\text{-D}(Q)(w)$ is defined only if a true proposition in the Hamblin set Q is stronger than every true proposition in Q; when defined, $\textsc{Ans}\text{-D}(Q)(w)$ returns this proposition.)

- Ans-H and Ans-D yield the same result if the Karttunen set of a question is closed under conjunction. Consider the yielded answers of the following questions:

(15) a. Is it raining?
b. Who formed the committee?

(16) a. Who came?
b. Which students came?

2.3.2 Explaining Uniqueness

- Dayal (1996) assumes the standard GB-transformed Karttunen Semantics. She defines *which*-phrases standardly as existential indefinites quantifying over the *wh*-complement:

(17) a. $[\![\text{which student}]\!] = \lambda f.\exists x[\mathsf{student}_{@}(x) \wedge f(x)]$
b. $[\![\text{which students}]\!] = \lambda f.\exists x[{}^{*}\mathsf{student}_{@}(x) \wedge f(x)]$

The Hamblin set yielded by a plural question (18a) is richer than the one yielded by its singular counterpart (18b): the former includes both singular answers and plural answers, while the latter consists of only singular answers.[2]

(18) (*w: Among the students, only Andy and Billy came.*)

a. Which students came?
$Q = \{{}^{\wedge}\mathsf{came}(x) \mid x \in {}^{*}\mathsf{student}_{@}\}$
$Q_w = \{{}^{\wedge}\mathsf{came}(a), {}^{\wedge}\mathsf{came}(b), {}^{\wedge}\mathsf{came}(a \oplus b)\}$
$\textsc{Ans}\text{-D}(Q)(w) = {}^{\wedge}\mathsf{came}(a \oplus b)$

b. Which student came?
$Q = \{{}^{\wedge}\mathsf{came}(x) \mid x \in \mathsf{student}_{@}\}$
$Q_w = \{{}^{\wedge}\mathsf{came}(a), {}^{\wedge}\mathsf{came}(b)\}$
$\textsc{Ans}\text{-D}(Q)(w)$ is undefined

To avoid this presupposition failure, (18b) can only be evaluated in a world where only one of the students came, which therefore explains its uniqueness requirement.

[2] Dayal (2017: §2.3.3) instead assumes that possible answers of a plural-marked questions are based on non-atomic individuals.

2.3.3 Other consequences of Dayal's presupposition

- **Discussion:** Dayal's presupposition also predicts the infelicity of the following. Can you see why?

(19) 'Which two students pass the exam? # I heard that three students did.'

- In addition to uniqueness, Dayal's presupposition also explains why constituent and polar questions do not combine well. (Bittner 1998)

(20) *Did which student/students came?
(*w: Among the students, only Andy came.*)
a. Singular
$Q_w = \{\hat{}\mathsf{came}(a), \hat{}\neg\mathsf{came}(b)\}$
b. Plural
$Q_w = \{\hat{}\mathsf{came}(a), \hat{}\neg\mathsf{came}(b), \hat{}\neg\mathsf{came}(a \oplus b)\}$

- Remaining issues: (We'll return to these issues in the coming weeks.)

1. Dayal's presupposition predicts an unwanted uniqueness effect for questions with a non-divisive collective predicate:

(21) Which students formed a team?
(*w: The students formed two teams in total: a+b formed one; c+d formed one.*)
a. $Q = \{\hat{}\mathsf{f.a.t.}(x) \mid x \in {}^*\mathsf{student}_@\}$
b. $Q_w = \{\hat{}\mathsf{f.a.t.}(a \oplus b), \hat{}\mathsf{f.a.t.}(c \oplus d)\}$
c. ANS-D$(Q)(w)$ is undefined (incorrect)

2. Ans-D always returns an exhaustive answer, which is too strong for mention-some questions.

(22) Jenny knows where we can get coffee nearby.

3. Negative island effects

3.1. Weak islands

- Negative island is usually considered as a type of weak islands. Weak islands are "weak" in the sense that they block extraction of adjuncts (*how/why*) but not arguments (*which NP*, *who/what*).

(23) Negative islands
  a. Who$_i$ didn't you talk to t_i?
  b. * How$_i$ didn't you behave t_i?
  c. * Why didn't you leave t_i?

(24) Factive islands
  a. ? Who did you find out that Mary had talked to t_i?
  b. * How did you find out that Mary had behaved t_i?
  c. * Why did you find out that Mary had left t_i?

(25) Interrogative islands
  a. ? Who did you wonder whether Mary had talked to t_i?
  b. * How did you wonder whether Mary had behaved t_i?
  c. * Why did you wonder whether Mary had left t_i?

It has been controversial whether the basis of weak island effects is syntactic (Huang 1982, Lasnik and Saito 1984, Chomsky 1986) or semantic (Szabolcsi and Zwarts 1993, a.o.). See summary and references in book Dayal (2017: chapter 6) and Abrusán (2014).

3.2. Negative island effects in degree questions

- It seems that degree questions are sensitive to negative islands: *how*$_D$-phrases cannot be extracted over negation.

(26) a. How much$_i$ does John weigh t_i?
  b. * How much$_i$ doesn't John weigh t_i?

Rullmann (1995): The ungrammaticality of (26b) is not due to syntax, but rather due to the semantic component this question cannot have an answer.

3.2.1 Rullmann (1995)

- A good answer must specify the **maximal degree**. A question is defined only if such a degree exists.

(27) How much/many ϕ?
  ≈ 'What is the maximal degree d s.t. $\phi(d)$?'

In (26b), as long as the set of degrees corresponding to weights has no upper bound, there can be no maximal degree d s.t. John doesn't weight d pounds.

(28) a. How much does John weigh?
  $?n[n = \textsc{max}(\lambda d(\text{John weighs } d\text{-pounds}))]$
  'What is the maximal degree d s.t. John weighs (at least) d pounds?'
  b. *How much doesn't John weigh?
  $?n[n = \textsc{max}(\lambda d(\neg\text{John weighs } d\text{-pounds}))]$
  '#What is the maximal degree d s.t. John doesn't weigh (at least) d pounds?'

– Also for comparatives with definites:

(29) a. * I have the amount of water that you don't.
b. I have the amount of water that you do
c. I have an amount of water that you don't.

– D-linking improves a negative degree question:

(30) Q: [Rob is a super athlete who has run international races of almost any distance.] How far has Rob never run?
A: He has never run a 25 mile race and he's never run a 100 mile race, (thought he has run all others, including a 50 mile race.)

Here degrees are not ordered on a linear scale; instead, they are ordinary entities structured by a Linkian part-of relation. This allows for an answer like which picks out a maximal element $25 \oplus 100$ to the exclusion of 50. (This is consistent with Szabolcsi and Zwarts' (1993) view of the role of D-linking on non-individual denoting expressions.)

- Problems: the maximal degree requirement is too strong.
  – Case 1: *sufficient*-questions (Beck and Rullmann 1999)
  (31a-b) are good, while $\{d \mid d$-much flour is sufficient to bake a cake$\}$ is not upper-bound.

  (31) a. How much flour is sufficient to bake a cake?
  b. I have the amount of flour that is sufficient to bake a cake.

  – Case 2: **Modal Obviation** (Kuno and Takami 1997; Fox and Hackl 2007)
  The ungrammaticality of a negative degree question is obviated if negation appears immediately above a possibility modal or below a necessity modal.

  (32) a. How fast is John not allowed to drive? (Neg $\gg \Diamond$)
  b. How fast is John required not to drive? ($\Box \gg$ Neg)

3.2.2 Beck and Rullmann (1999)

- A good answer must specify the degree that yields the **maximally informative (max-inf) true answer** (*pace* Dayal's presupposition). As such, a question is defined only if such a degree exists.

(33) How much/many ϕ?
≈ 'What is the degree d that yields the max-informative among the true propositions of the form $\phi(d)$?

However, the basic explanation for the negative island effect is lost. Suppose that John's weight is just below 150 pounds, 'John doesn't weigh 150 pounds' is the max-inf true answer. (Conversely, if John weighs 150 pounds, 'John doesn't weigh d' (where d is the degree right above 150 pounds) is the max-inf true answer.

(26b) * How much$_i$ doesn't John weigh t_i?

3.2.3 Density of scales (Fox and Hackl 2007)

- **Explaining a basic negative island**

– Fox and Hackl adopt the max-informativity requirement but further assume *universal density of measurement (UDM)*: natural language semantics treats all measurements uniformly as mappings from objects (individuals or collections of individuals) to dense scales.

(34) **Universal density of measurement (UDM)**
$\forall d_1 d_2[d_1 < d_2 \rightarrow \exists d_3[d_1 < d_3 < d_3]]$
(For any two degrees d_1 and d_2 in a given scale, there is a degree d_3 between d_1 and d_3.)
(E.g.: the set of rational numbers (**Q**) is densely ordered, while the set of integers (**Z**) is not.)

Example:

(26b) * How much$_i$ doesn't John weigh t_i?

The nucleus 'not [John weights t_i]' is downward-entailing w.r.t. t_i; thus, the max-informative true answer, if it exists, should be based on the **minimal** degree d_{min} s.t. John doesn't weigh d_{min}. Due to density, however, d_{min} does not exist:

> Proof: Assume that John weighs exactly d_0. Then for any degree $d_0+\epsilon$, however small ϵ is, there is a degree d' such that $d_0 < d' < d_0+\epsilon$ and John doesn't weigh d'.

– Why saying that measurement is 'universally' dense?

(35) * How many children doesn't John have?

To predict this ungrammaticality based on max-informativity, the set of numbers used to measure the quantity of children shall be dense, not the set of integers. Otherwise, if John has n children, 'John doesn't have n+1 children' would be the max-inf true answer of (35).

- **Modal obviation**

(36) How much are you sure that this vessel won't weigh?

Though there can be no minimal degree d that the vessel doesn't weigh, there can be a minimal weight d_{min} s.t. in every world of the modal base (the set of worlds that represent the addressee's certainty about the vessel's weight), the vessel doesn't weigh d_{min} in that world. This d_{min} names the max-inf true answer.

Discussion: Following F&H's account, think about why the following sentences are (un)grammatical.

(37) a. How fast is John required not to drive? ($\Box \gg$ Neg)
b. * How fast is John allowed not to drive? ($\Diamond \gg$ Neg)

3.2.4 The interval-based account (Abrusán and Spector 2011)

- Abrusán and Spector also make use of Dayal's presupposition. But unlike F&H, they propose that a scalar predicate ranges over **intervals** of degrees.

(38) a. $[\![\text{tall}]\!] = \lambda D : D$ is an interval. $\lambda x[x\text{'s height} \in D]$

b. Given a scale E (a set ordered by $\leq$), D is an interval on E iff $D \subseteq E$ and $\forall d_1 d_2 d_3 \in E[[d_1 d_3 \in D \wedge d_1 \leq d_2 \leq d_3] \rightarrow d_2 \in D]$

(39) a. How tall is John? ≈ 'For what interval I, John's height belongs to I?'
Let John's exact height in w be 5ft, then:

Q_w =
- ☞ John's height belongs to [5, 5]
- John's height belongs to [1, 5]
- John's height belongs to [2, 6]
- ...
- ~~John's height belongs to [0, 5)~~
- ~~John's height belongs to (5, +∞]~~
- ...

b. *How tall isn't John? ≈ 'For what interval I, John's height doesn't belong to I?'
Let John's exact height in w be 5ft, then:

Q_w =
- ~~John's height doesn't belong to [5, 5]~~
- ~~John's height doesn't belong to [1, 5]~~
- ~~John's height doesn't belong to [2, 6]~~
- ...
- ? John's height doesn't belong to [0, 5)
- ? John's height doesn't belong to (5, +∞]
- ...

NB: In this account, there is no need to assume density of measurement. A&S argue that this provides a better account of non-degree questions, while preserving the modal obviation facts along the lines proposed by F&H.

- Modal obviation

(40) a. How fast are we not allowed to drive?

b. How fast are we required not to drive?

c. Let the speed limit in w be 65mph, then:

Q_w =
- ~~We are not allowed to drive [64mph, 64mph]~~
- ~~We are not allowed to drive [65mph, 65mph]~~
- ☞ We are not allowed to drive [66mph, 66mph]
- We are not allowed to drive [67mph, 67mph]
- ...

References

Abrusán, Márta. 2014. *Weak island semantics*, volume 3. OUP Oxford.

Abrusán, Márta, and Benjamin Spector. 2011. A semantics for degree questions based on intervals: Negative islands and their obviation. *Journal of Semantics* 28:107–147.

Beck, Sigrid, and Hotze Rullmann. 1999. A flexible approach to exhaustivity in questions. *Natural Language Semantics* 7:249–298.

Bittner, Maria. 1998. Cross-linguistic semantics for questions. *Linguistics and Philosophy* 21:1–82.

Dayal, Veneeta. 1996. *Locality in Wh Quantification: Questions and Relative Clauses in Hindi*. Dordrecht: Kluwer.

von Fintel, Kai, Danny Fox, and Sabine Iatridou. 2014. Definiteness as maximal informativeness .

Fox, Danny, and Martin Hackl. 2007. The universal density of measurement. *Linguistics and Philosophy* 29:537–586.

Heim, Irene. 1994. Interrogative semantics and Karttunen's semantics for *know*. In *Proceedings of IATOML1*, volume 1, 128–144.

Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice-theoretical approach. In *Meaning, use, and interpretation of language*, ed. Christoph Schwarze Rainer Bäuerle and Arnim von Stechow, 302–323. De Gruyter.

Rullmann, Hotze. 1995. Maximality in the semantics of *wh*-constructions. Doctoral Dissertation, University of Massachusetts at Amherst.

Rullmann, Hotze, and Sigrid Beck. 1998. Presupposition projection and the interpretation of 'which'-questions. In *Semantics and linguistic theory*, volume 8, 215–232.

Sauerland, Uli, Jan Anderssen, and Kazuko Yatsushiro. 2005. The plural is semantically unmarked. *Linguistic evidence* 413–434.

Sharvy, Richard. 1980. A more general theory of definite descriptions. *The philosophical review* 89:607–624.