Partition Semantics

1. Requirements of interpreting interrogatives

1.1. Belnap's (1981) Requirements

- Belnap (1982) provides three general methodological constraints for classifying and evaluating different theories of questions and answers.

  1. **Independent meaning requirement**: Interrogatives are entitled to a meaning of their own.
     This requirement goes against theories that analyze interrogatives as performatives (Lewis 1972; Krifka 2001) or as epistemic imperatives (Hintikka 1976).

     (1) Does Mary walk in the garden?
        a. I (hereby) ask you whether Mary walks in the garden — performatives
        b. Bring it about that I know whether Mary walks in the garden — epist. imperatives

  2. **Equivalence requirement:** (Matrix) interrogatives and their embedded forms are to be treated on a par (i.e., semantically intimately related, or even identical).
  3. **Answerhood requirement:** The meaning of an interrogative resides in its answerhood conditions.

1.2. World-(in)dependency

- Some question-embeddings are **index/world-dependent**, as in (2); while some others are not, as in (3).

(2) Andy knows/tells whether it is raining.
   a. If it is raining, Andy knows/tells that it is raining.
   b. If it isn't raining, Andy knows/tells that it isn't raining.

(3) Andy wonders whether it is raining.

1.3. Coordination and entailments

- Conjunction/disjunction is standardly defined as set **intersection/union**. Interestingly, even matrix interrogatives can be coordinated.

(4) a. Who went out for a walk? And who stayed home?
   b. Who went out for a walk? And did they take the dog along?
   c. Does John walk? Or does Mary walk?

Moreover, interrogatives can also be coordinated with declaratives in embeddings:

(5) a. John knows that Peter has left for Paris, and also whether Mary has followed him.
   b. Alex told Susan that someone was waiting for her, but not who it was.

- **Entailment** is standardly defined as **set inclusion**. Though, descriptively, one can say that an interrogative Q_1 entails Q_2 iff every proposition that gives a complete true answer to Q_1 also gives such an answer to Q_2.

(6) a. John walked.
John talked.
———————
John walked and talked.

b. Which men walked?
Which men talked?
———————
Which men walked and talked?

(7) a. Only Mary walks in the park.
———————
John doesn't walk in the park.

b. Who walks in the park?
———————
Does John walk in the park?

- Gr&S's arguments against competing approaches

– Categorial approaches do not assign interrogatives with a uniform interpretation, which makes it hard to get coordinations of interrogatives (of the same or different categories) and entailments between interrogatives.

– Karttunen Semantics doesn't assign the right type of semantic object to interrogatives.

(i) Question coordination cannot be treated as intersection of the Karttunen sets of the coordinated questions. For example, (4a-b) would denote empty sets.

(ii) Entailment is standardly defined as set inclusion. Interpreting interrogatives as denoting sets of propositions implies that Q_1 entails Q_2 iff $[\![Q_1]\!] \subseteq [\![Q_2]\!]$. But entailment between questions cannot be thought of as inclusion between the Karttunen sets of these questions:

(8) Who walks in the park?
———————
Does John walk in the park?

a. {John walks in the park, Mary walks in the park, J+M walk in the park }.
———————
{John walks in the park}. (Invalid)

b. {Mary walks in the park}.
———————
{John doesn't walk in the park}. (Invalid)

2. Partition Semantics of questions

The three works on Partition Semantics by Groenendijk and Stokhof (1982, 1984, 1989) involve different assumptions (mostly on technicalities). We'll follow mostly Gr&S (1984) on intensional-vs-extensional predicates, and Gr&S (1989) on question coordinations. Techniques for Ty2 (viz., the language of two-sorted type theory) are removed for simplicity.

2.1. Defining questions

- Indicatives versus interrogatives

(9) a. Extension: *truth value*
$[\![\text{it is raining}]\!]^w = 1$ iff it is raining in w

b. Intension: *proposition*
$[\![\text{whether it is raining}]\!] = \lambda w.[\![\text{it is raining}]\!]^w = 1$

(10) a. Extension: *proposition*
$[\![\text{whether it is raining}]\!]^w = \lambda w'[\mathsf{rain}(w) = \mathsf{rain}(w')]$

b. Intension: *propositional concept (i.e., partition)*
$[\![\text{whether it is raining}]\!] = \lambda w \lambda w' [\mathsf{rain}(w) = \mathsf{rain}(w')]$
(Or equivalently, a relation between indices: $[\![?\phi]\!] = \{\langle w, w' \rangle \mid [\![\phi]\!]^w = [\![\phi]\!]^{w'}\}$)

- Given a set of possible worlds W, a partition of W is made up of a set of non-overlapped cells such that the union of this cells equals to W. Two worlds belong to the same cell of a partition if and only if the question nucleus has the same extension in these two worlds.

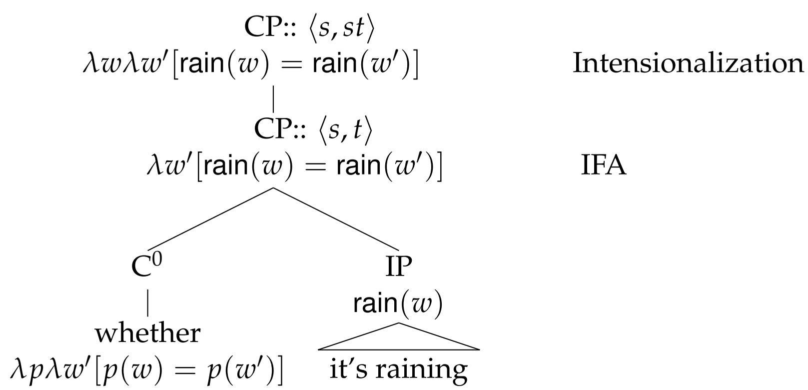
w: $\mathsf{rain}(w) = 1$	=	w: it is raining in w
w: $\mathsf{rain}(w) = 0$		w: it is not raining in w

Table 1: Partition for *whether it is raining*

2.2. Composing polar questions

- Composing via a GB-style LF, we can say that *whether*/$O_{\mathrm{Y/N}}$ combines with the question nucleus via Intensional Functional Application (IFA).

(11) Is it raining?



2.3. Combining with an embedding verb (Gr&S 1984: chapter 2)

- Verbs like *know* and *tell* are **extensional**. They take the extension of an embedded interrogative (i.e., a proposition) as argument.

(12) John **knows/tells** whether it's raining.
  a. If it's raining, then J knows/tells that it's raining.
  b. If it isn't raining, then J knows/tells that it isn't raining.

Explain index-dependency:

(13) $\mathsf{know}(w)(j, \lambda w' [\mathsf{rain}(w) = \mathsf{rain}(w')])$
  a. If it rains in w, then: $[\![(12)]\!]^w = \mathsf{know}(w)(j, \lambda w' [\mathsf{rain}(w') = 1])$
  b. If it doesn't rain in w, then: $[\![(12)]\!]^w = \mathsf{know}(w)(j, \lambda w' [\mathsf{rain}(w') = 0])$

- Verbs like *wonder* and *guess* are **intensional**. They take the intension of an embedded interrogative (i.e., a partition) as argument.

(14) John wonders whether it is raining.
$\mathsf{wonder}(w)(j, \lambda w \lambda w' [\mathsf{rain}(w) = \mathsf{rain}(w')])$

This assumption explains why question-embeddings with *wonder* isn't world-dependent, and why *wonder* cannot select a declarative complement.

(15) John knows/*wonders that it is raining.

2.4. *Wh*-questions

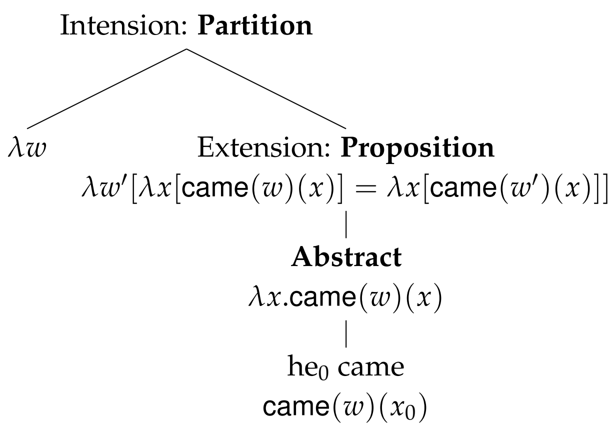
2.4.1 Forming a partition

- In a *wh*-question, the formation of a question denotation involves three steps:

  1. The same as in categorial approaches, *wh*-terms abstract out the corresponding variables from the question nucleus, forming a **λ-abstract**. In Gr&S's treatment, *wh*-terms do not belong to a fixed syntactic category; they are like the λ-abstraction sign in the logical language counterpart.
  2. Type-shifting this λ-abstract yields a set of worlds (a **proposition**) s.t. the λ-abstract holds for the same set of individuals in these worlds and in the evaluation world.
  3. Intensionalizing the above proposition yields a **propositional concept**, i.e., a **partition of possible worlds**.

Example: The formation of the partition of 'who came':

(16)



Consider only two individuals John and Mary, the partition can be represented as:

w: $\lambda x[\mathsf{came}(w)(x)] = \{j, m, j \oplus m\}$	w: $\lambda x[\mathsf{came}(w)(x)] = \{m\}$
w: $\lambda x[\mathsf{came}(w)(x)] = \{j\}$	w: $\lambda x[\mathsf{came}(w)(x)] = \varnothing$

=

w: only j and m came in w	w: only m came in w
w: only j came in w	w: nobody came in w

Table 2: Partition for *who came*

Exercise: Write out the λ-abstract and the partition of the following multi-*wh* question. Consider only two individuals *ab*, illustrate this partition with a table.

(17) Which person voted for which person?

2.4.2 Exhaustivity in indirect *wh*-questions

- Two forms of exhaustivity (discussed by Gr&S):

(18) John knows who came.
If x came, J bels that x came. **Weak**
If x came, J bels that x came; if x didn't come, J bels that x did**n't** come. **Strong**

- A partition denoted by a question is a function from a world w to the **strongly exhaustive** answer of this question in w. Hence, knowing a question amounts to knowing the strongly exhaustive answer of this question.

(19) Andy knows who came.
$\mathsf{know}(w)(a, \lambda w'[\lambda x[\mathsf{came}(w)(x)] = \lambda x[\mathsf{came}(w')(x)]])$
  a. If only Mary came in w, then:
  $[\![(19)]\!]^w = \mathsf{know}(w)(a, \lambda w'[\lambda x[\mathsf{came}(w')(x)] = \{m\}])$
  b. If only John and Mary came in w, then:
  $[\![(19)]\!]^w = \mathsf{know}(w)(a, \lambda w'[\lambda x[\mathsf{came}(w')(x)] = \{j, m, j \oplus m\}])$

Gr&S claim that the strong exhaustiveness their theory delivers is required.[1]

(20) John believes that Bill and Suzy walk.
Only Bill walks.
———
John doesn't know who walks.

Discussion: Assume that it is raining and that only Mary came, are the sentences in each pair predicted to be semantically equivalent under Partition Semantics? Why or why not?

(21) a. John knows that it is raining.
  b. John knows whether it is raining.

(22) a. John knows that Mary came.
  b. John knows who came.

(23) a. John knows who came.
  b. John knows who didn't come.

Discussion: How does Partition Semantics predict following entailment between questions?

(24) a. Andy knows who walks in the park.
  ———
  Andy knows whether John walks in the park.
  b. Who walks in the park?
  ———
  Does John walk in the park?

[1] Later works show that strong exhaustivity is too strong. We will return to this issue later.

2.4.3 The *de dicto*/*de re* ambiguity

- Is the following argument valid?

  (25) John knows who walks.
  ―――――――――――――――
  John knows which student walks.

  It depends on how the conclusion is read. The argument is ...

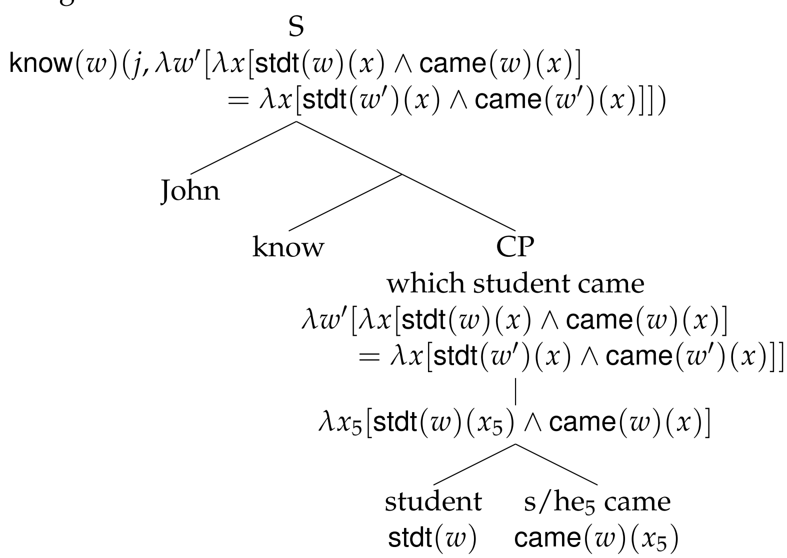
  ... **valid** if the conclusion is taken *de re*. For any w: $[\![\text{student}]\!]^w \subseteq [\![\text{hmn}]\!]^w$. Hence, if one knows of a set which of its elements have a certain property, one also knows this of every subset of that set.

  ... **invalid** if the conclusion is taken *de dicto*. Suppose that the person who walks is a student and John knows that only s/he walks; but John fails to know that s/he is a student.

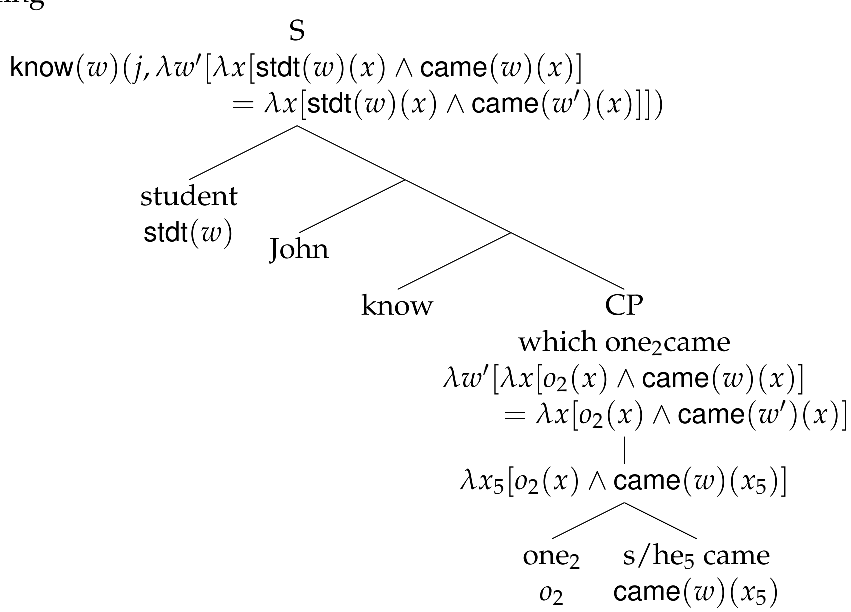
- Deriving the ambiguity (ignore number-marking)

  (26) John knows which student came.

  a. *de dicto* reading



  b. *de re* reading



- **Discussion:** The *de dicto/de re* ambiguity also matters for the validity of the following argument. Can you tell why?

(27) John knows which student walks.
―――――――――――――――
John knows which student doesn't walk.

3. Coordination of questions (Groenendijk and Stokhof 1989)

- For an embedded question coordination, each coordinated questions are assigned with a "lifted" interpretation.[2] To combine with a meaning of a complex type, the question-embedding predicate is also type-lifted via object-raising.

(29) $[\![A \text{ and } B]\!] = A' \sqcap B'$
$$= \begin{cases} A' \wedge B' & \text{if } A' \text{ and } B' \text{ are of type } t \\ \lambda x[A'(x) \sqcap B'(x)] & \text{if } A' \text{ and } B' \text{ are of some other conjoinable type} \\ \text{undefined} & \text{otherwise} \end{cases}$$

(30) Jenny knows whether ϕ and whether ψ.

a. $[\![\text{whether } \phi \text{ and whether } \psi]\!]^w = [\![\text{whether } \phi]\!]^{w,\Uparrow_{\text{LIFT}}} \sqcap [\![\text{whether } \psi]\!]^{w,\Uparrow_{\text{LIFT}}}$
$= \lambda R_{\langle st,t\rangle}[R(\lambda w'[[\![\phi]\!]^w = [\![\phi]\!]^{w'}])] \sqcap \lambda R[R(\lambda w'[[\![\psi]\!]^w = [\![\psi]\!]^{w'}])]$
$= \lambda R_{\langle st,t\rangle}[R(\lambda w'[[\![\phi]\!]^w = [\![\phi]\!]^{w'}]) \wedge R(\lambda w'[[\![\psi]\!]^w = [\![\psi]\!]^{w'}])]$

b. $[\![\text{know}]\!]^{\Uparrow_{\text{OBJ}}} = \lambda Q_{\langle\langle st,t\rangle,t\rangle}\lambda x_e[Q(\lambda p_{st}.\mathsf{know}(w)(x,p))]$

c. $[\![\text{know}]\!]^{\Uparrow_{\text{OBJ}}}([\![\text{whether } \phi \text{ and whether } \psi]\!]) = ...$

- The challenging case: disjunctions embedded under *wonder*

(31) John wonders who walks or who talks.

a. John wants to know who walks or to know who talks. Narrow (non-distributive)

b. John wants to know who walks or he wants to know who talks. Wide (Distributive)

  - Gr&S (1984) treat *wonder* an intensional predicate (of type $\langle\langle st,t\rangle, et\rangle$) that takes a propositional concept as argument. However, this treatment yields only the distributive reading.
  - To get the non-distributive reading, *wonder* should combine with the intension of the disjunction. Gr&S (1989) alternatively define *wonder* with a higher type $\langle\langle s, \langle stt, t\rangle\rangle, et\rangle$). To get back to the distributive reading, one can first apply argument-lowering and then argument-lift to get back to the higher type of *wonder*. Roughly:

(32) $([\![\text{wonder}]\!]^{\Downarrow}(\lambda w.[\![\text{Q}_1 \text{ or } \text{Q}_2]\!]^w))^{\Uparrow}$

[2] Gr&S (1984: chapter 2) instead define each coordinated question as a **set of properties of propositional concepts**. A question coordination is thus the intersection/meet of such property sets.

(28) a. $[\![\text{whether } \phi]\!] = \lambda w'[[\![\phi]\!]^w = [\![\phi]\!]^{w'}]$

b. $[\![\text{whether } \phi]\!]^{\uparrow} = \lambda R_{\langle s,\langle stt,t\rangle\rangle}[R(w)(\lambda w \lambda w'[[\![\phi]\!]^w = [\![\phi]\!]^{w'}])]$

4. Comparing the three denotations

- So far, there have been three types of denotations proposed to be the root denotations of questions:

(33) Who came?

a. $\mathbf{P} = \lambda x[\mathsf{hmn}_{@}(x) = 1.\hat{}\mathsf{came}(x)]$ — λ-abstract (Topical property)

b. $Q = \{\hat{}\mathsf{came}(x) : \mathsf{hmn}_{@}(x) = 1\}$ — Hamblin set

c. $\textsc{Part} = \lambda w \lambda w'[\lambda x[\mathsf{hmn}_{@}(x) \wedge \mathsf{came}(w)(x)] = \lambda x[\mathsf{hmn}_{@}(x) \wedge \mathsf{came}(w')(x)]]$ — Partition

	Topical property	Hamblin set	Partition
Retrieving the question nucleus	Yes	No	No
Getting constituent answers	Yes	No	No
Getting weakly exhaustive answers	Yes	Yes	No
Getting strongly exhaustive answers	Yes	Yes	Yes
Uniform semantic type	No	Yes: $\langle st, t\rangle$	Yes: $\langle s, stt\rangle$

The three denotations can be ranked as follows ('A > B' means that any information that is derivable from meaning B is also derivable from meaning A, but not the other direction):

Lambda abstracts (Categorial)	>	Hamblin sets (Hamblin-Karttunen)	>	Partitions (Partition)

For example, starting from a λ-abstract, we can derive all the meanings that are derivable from a Hamblin set or a partition, but not in the other direction. The reason is that Hamblin sets and partitions can be defined based on a λ-abstract, while λ-abstracts cannot be retrieved from Hamblin sets or partitions.

4.1. Lambda abstracts > Hamblin sets

- From λ-abstracts to Hamblin sets: EASY

(34) $Q = \{\mathbf{P}(\alpha) \mid \alpha \in \mathrm{Dom}(\mathbf{P})\}$
(The set of propositions obtained by applying **P** to its possible arguments.)

- From Hamblin sets to λ-abstracts: DIFFICULT

(35) a. Did JOHN come or MARY come?$_{\text{ALT-Q}}$
b. [Among John and Mary,] which person came?

The following two different λ-abstracts yield the same Hamblin set (i.e., $\{f(a), f(b)\}$). Hence, given a Hamblin set, we cannot retrieve the λ-abstracts, nor the short answers.

(36) a. $\mathbf{P}_1 = \lambda p : p \in \{f(j), f(m)\}.p$
b. $\mathbf{P}_2 = \lambda x : x \in \{j, m\}.f(x)$

4.2. Lambda abstracts & Hamblin sets > Partitions

- From λ-abstracts to partitions: EASY

(37) $\lambda w \lambda w' [\mathbf{P}_w = \mathbf{P}_{w'}]$ $\qquad$ $\mathbf{P}_w$ stands for $\lambda x[\mathbf{P}_w(x) = 1]$

From Hamblin sets to partitions: EASY

(38) $\lambda w \lambda w' [Q_w = Q_{w'}]$ $\qquad$ Q_w stands for $\{p \mid p(w) = 1 \wedge p \in Q\}$

- From partitions to Hamblin sets and λ-abstracts: DIFFICULT

The following two questions have different λ-abstracts and Hamblin sets: *which person* only quantifies over atomic elements, while *who* quantifies over also sums (as well as generalized boolean conjunctions/disjunctions)

(39) a. Who came?

$\mathbf{P} = \lambda x[\mathsf{hmn}_@(x) = 1.\hat{}\mathsf{came}(x)]$

$$Q = \left\{ \begin{array}{l} \hat{}\mathsf{came}(j) \\ \hat{}\mathsf{came}(m) \\ \hat{}\mathsf{came}(j \oplus m) \end{array} \right\}$$

b. Which person came?

$\mathbf{P} = \lambda x[\mathsf{person}_@(x) = 1.\hat{}\mathsf{came}(x)]$

$$Q = \left\{ \begin{array}{l} \hat{}\mathsf{came}(j) \\ \hat{}\mathsf{came}(m) \\ \end{array} \right\}$$

But they have the same partition:

Partition yielded by (39a)				Partition yielded by (39b)
w: $\{x : w \in c(x)\} = \{j, m, j \oplus m\}$	=	w: only $j \oplus m$ came in w	=	w: $\{x : w \in c(x)\} = \{j, m\}$
w: $\{x : w \in c(x)\} = \{j\}$		w: only j came in w		w: $\{x : w \in c(x)\} = \{j\}$
w: $\{x : w \in c(x)\} = \{m\}$		w: only m came in w		w: $\{x : w \in c(x)\} = \{m\}$
w: $\{x : w \in c(x)\} = \varnothing$		w: nobody came in w		w: $\{x : w \in c(x)\} = \varnothing$

4.3. Exercise

- In the following, do the questions in each pair have the same λ-abstract/ Hamblin set/ partition?

(40) a. Who came?
b. Who didn't come?
c. Only which people came?

(41) a. Did John arrive?
b. Did John arrive or not?

- Do the following questions have the same partition?

(42) a. Which people x is such that only x came?
b. Which person x is such that only x came?

- Based on a Karttunen set of a question, is it possible to derive the λ-abstract/ Hamblin set/ partition of this question?

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