Intensional Semantics

1. Why extensional semantics is insufficient — the Opacity Problem

- In Extensional Semantics, every expression is of type e, or t, or a derived type based on e and t. The meaning of a complex expression is composed from the extension of each component of this complex expression.

  Leibniz's law: If two expressions have the same denotation, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same.

  (1) (w: Andy is the chair. Andy invited Betty.)

  a. $[\![\text{Andy invited Betty}]\!]^w$
  $= [\![\text{invited}]\!]^w([\![\text{Betty}]\!]^w)([\![\text{Andy}]\!]^w)$
  $= \textsf{invited}_w(a, b)$
  $= 1$

  b. $[\![\text{The chair invited Betty}]\!]^w$
  $= [\![\text{invited}]\!]^w([\![\text{Betty}]\!]^w)([\![\text{the chair}]\!]^w)$
  $= \textsf{invited}_w(\iota x.\textsf{chair}_w(x), b)$
  $= \textsf{invited}_w(a, b)$
  $= 1$

- But, if we consider only the extension of an expression, there are counterexamples of Leibniz's law.

– **Example 1** (from Coppock & Champollion 2018: chap. 10)

The Morning Star happens to be the Evening Star. (Both name Venus.) But, by intuition, there is a reading under which the sentence (a) is true and (b) is false.

(2) a. Necessarily, the Morning Star is the Morning Star.
b. Necessarily, the Morning Star is the Evening Star.

Contexts in which the substitution of one term for a co-referential term can lead to a change in truth value are called **referentially opaque contexts**.

– **Example 2** (from Winter 2016 *Elements of Formal Semantics*: chap. 5)

Assume that every pianist is a composer in w, and that every composer is a pianist w. Then:

(3) $[\![\text{composer}]\!]^w = [\![\text{pianist}]\!]^w$

For sure, (4a)/(5a) has the same truth value as (4b)/(5b) in w.

(4) a. In Tina's room, some composer is playing.
b. In Tina's room, some pianist is playing.

(5) a. John is talking to a pianist.
b. John is talking to a composer.

But, (6a)/(7a) might not have the same truth value as (6b)/(7b) in w.

(6) a. In Tina's dream, some composer is playing.
b. In Tina's dream, some pianist is playing.

(7) a. John is looking for a pianist.
b. John is looking for a composer.

In (6), it might be the case that no composer is a pianist in Tina's dream. In (7), John may be looking for a pianist without being aware that pianists are composers.

Sentences (7a-b) are ambiguous between a ***de re*** reading and a ***de dicto*** reading. On a *de re* interpretation, (7a) means that there is a particular pianist such that John is looking for it. For example: John is looking for Mary, and Mary happens to be a pianist. On a *de dicto* interpretation; it means that Jane is looking for a pianist, whichever pianist.

- **Example 3**

Assume that Gennaro smokes and likes Belgian chocolate in w. Then:

(8) $[\![\text{Gennaro smokes}]\!]^w = [\![\text{Gennaro likes Belgian chocolate}]\!]^w = 1$

If the attitude predicate *believe* expresses a relation between the extension of the belief holder and the the extension of the embedded sentence, we have:

(9) $[\![\text{Kate believes S}]\!]^w = [\![\text{believe}]\!]^w([\![\text{S}]\!]^w)([\![\text{Kate}]\!]^w)$.

Then by Leibniz's law, the equation (8) yields the following equation:

(10) $[\![\text{Kate believes Gennaro smokes}]\!]^w = [\![\text{Kate believes Gennaro likes Belgian chocolate}]\!]^w$
a. $[\![\text{believe}]\!]^w([\![\text{Gennaro smokes}]\!]^w)([\![\text{Kate}]\!]^w) = \mathsf{believe}_w(k, 1)$
b. $[\![\text{believe}]\!]^w([\![\text{Gennaro likes Belgian chocolate}]\!]^w)([\![\text{Kate}]\!]^w) = \mathsf{believe}_w(k, 1)$

However, equation (10) doesn't hold; in fact (in world w), Kate knows that Gennaro smokes, but she doesn't know that he likes Belgian chocolate. Hence, *believe* doesn't express a relation between an individual/entity (the extension of the belief holder) and a truth value (the extension of the embedded clause).

- **Intensional expressions** (as opposed to **extensional expressions**):
  - predicates: *look for*, *believe*, *know*, *wish*, *doubt*, *hope*, *fear*, *think*, *discover*, *want*, *demand*, *expect*, ...
  - adverbials: *in Tina's dream*, *possibly*, *necessarily*, ...
  - modal verbs: *can*, *have to*, *should*, ...

Exercise: (i) Show that the sentence-embedding verb *know* is an intensional expression. (ii) Is the sentence-embedding verb *tell* intensional or extensional?

Exercise: The following sentence is ambiguous. What readings do you get?

(11) Suzi wants to marry a plumber.

2. Defining extension and intension

- The **extension** of a world-dependent expression is evaluated relative to a particular possible world. We add an *evaluation world parameter* $[\![\bullet]\!]^w$ to the notations of extensions:

  (12) General notation: $[\![X]\!]^w$ ('the extension of X in w')

- The **intension** of an expression X is a function which (i) takes a possible world as an argument, and (ii) returns the extension of X in that world.

  (13) General notation: $\lambda w_s.[\![X]\!]^w$ ('the intension of X')

  – The intension of a sentence is a function from worlds to truth values, called *proposition*.
  – The intension of a one-place predicate (IV/VP/NP/Pred Adj/...) of type $\langle e, t\rangle$ is a function from worlds to $\langle e, t\rangle$ functions, called *property*.
  – The intension of a definite DP is a function from worlds to entities, called *individual concept*.

  <u>Examples:</u>

  (14) Gennaro smokes.
  a. extension: $\mathsf{smokes}_w(g) = 1$ iff Gennaro smokes in w
  b. intension: $\lambda w_s[\mathsf{smokes}_w(g) = 1]$

  (15) composer
  a. extension: $\lambda x_e.\mathsf{composer}_w(x)$
  b. intension: $\lambda w_s \lambda x_e.\mathsf{composer}_w(x)$

  (16) the chair
  a. extension: $\iota x.\mathsf{chair}_w(x)$
  b. intension: $\lambda w_s.\iota x.\mathsf{chair}_w(x)$

- Using set-theoretic notations, a proposition can also be viewed as the set of possible worlds where this proposition is true.

  (17) The intension of "Gennaro smokes": $\{w : [\![\text{Gennaro smokes}]\!]^w = 1\}$

Relations and operations	Set-theoretical notations
p entails q	$p \subseteq q$
p contradicts q	$p \cap q = \varnothing$
p and q	$p \cap q$
p or q	$p \cup q$
p is possible	$p \neq \varnothing$
p is necessary	$p = W$

- **Discussions:** the following formulas are problematic. Identify and correct the problems.

  (18) a. $\forall q \in C[q \rightarrow q \subseteq p]$
  (Every true proposition in C entails p.)
  b. $(p \subseteq q) \rightarrow \neg q$
  (If p entails q, then q is false.)

3. Intensional-izing the theory of types and compositions

- Types and domains

(19) a. Basic types: e (individuals/entities) and t (truth values).
b. Functional types: If σ and τ are types, then $\langle \sigma, \tau \rangle$ is a type.
c. Intensional types: If σ is a type, then $\langle s, \sigma \rangle$ is an intensional type.

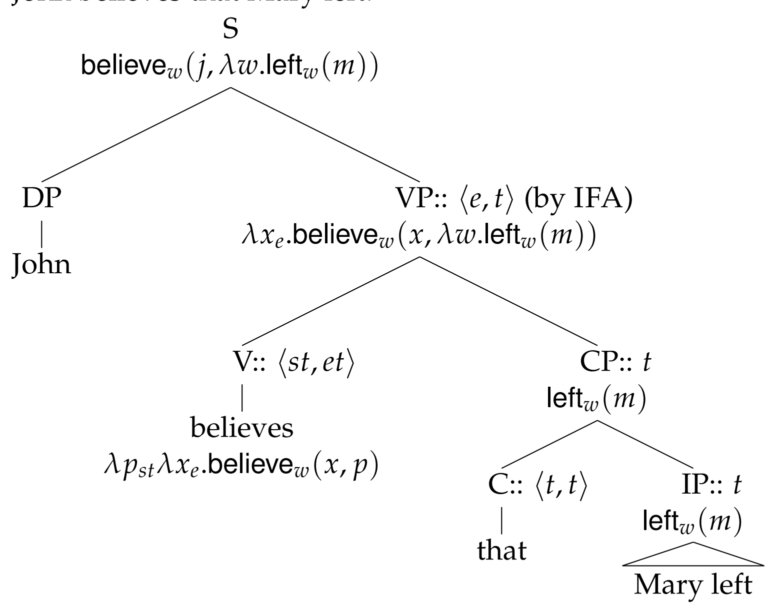
(20) a. $D_s = W$
b. $D_{\langle \sigma,\tau \rangle} = \{f \mid f : D_\sigma \to D_\tau\}$ (functions from things of type σ to things of type τ)
c. $D_{\langle s,\tau \rangle} = \{f \mid f : W \to D_\tau\}$ (functions from possible worlds to things of type τ)

- Composing with intensional expressions

– Approach I: new composition rule

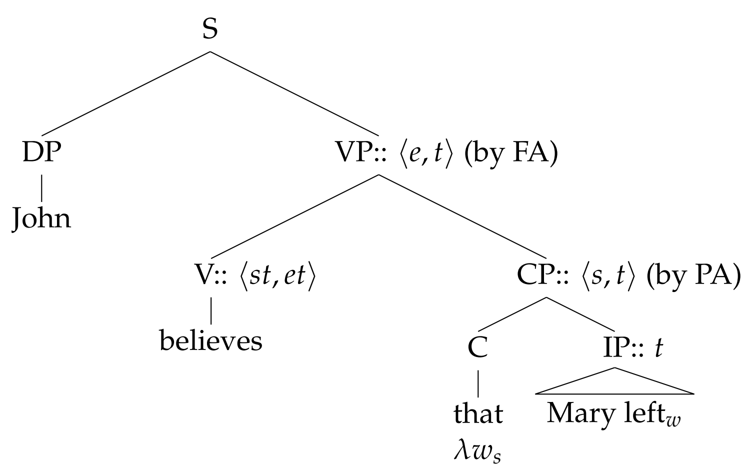
(21) **Intensional Functional Application (IFA)**
If $\{\beta, \gamma\}$ is the set of α's daughters, $[\![\beta]\!] \in D_{\langle\langle s,\sigma\rangle,\tau\rangle}$, and $[\![\gamma]\!] \in D_\sigma$, then $[\![\alpha]\!] = [\![\beta]\!](\lambda w_s.[\![\gamma]\!]^w)$

(22) John believes that Mary left.



– Approach II: we can assume that the verb *left* carries an world variable w, which is then abstracted over by a λ-operator. (See Percus 2000 "Constraints on some other variables in syntax".)

(23)



4. Defining *believe*

- The meaning of the clausal object of *believe* must be intensional.

(24) $[\![\text{Kate believes S}]\!]^w = [\![\text{believe}]\!]([\![\text{Kate}]\!]^w, \lambda w.[\![\text{S}]\!]^w)$

- "x believes S" is true in w iff S is true in every world that is compatible with x's belief in w, formally:

(25) a. $[\![\text{x believe S}]\!]^w = 1$ iff $\text{BELIEF}(x, w) \subseteq \{w' : [\![\text{S}]\!]^{w'} = 1\}$
b. $[\![\text{believe}]\!]^w = \lambda p_{st}.\lambda x_e.\text{BELIEF}(x, w) \subseteq p$

  - Consider the following four worlds:

    $w1$: Gennaro smokes, and he like Belgian chocolate.

    $w2$: Gennaro smokes, but he doesn't like Belgian chocolate.

    $w3$: Gennaro doesn't smoke, but he likes Belgian chocolate.

    $w4$: Gennaro doesn't smoke, and he doesn't like Belgian chocolate.

  Then:

  The intension of "Gennaro smokes": $\{w1, w2\}$
  The intension of "Gennaro likes Belgian chocolate": $\{w1, w3\}$

  - Assume that in the actual world w, Kate believes that Gennaro smokes, and she has no idea whether he likes Belgian chocolate. Then the worlds that are compatible with Kate's belief in w are $\{w1, w2\}$. Then "Kate believes S" is true in w iff S is true in both $w1$ and $w2$.
  - Now compute the extension of the two *believe*-sentences:

(26) a. $[\![\text{Kate believes Gennaro smokes}]\!]^w = 1$
(because for every world in $\{w1, w2\}$, Gennaro smokes is true.)
b. $[\![\text{Kate believes Gennaro likes Belgian chocolate}]\!]^w = 0$
(because there is a world in $\{w1, w2\}$ where Gennaro likes Belgian chocolate isn't true.)