Basics of composition

1. Composition with simple predicates

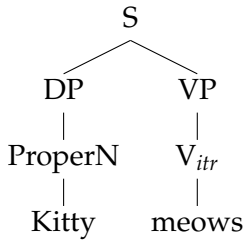
1.1. Basics

- **The principle of compositionality (Fregean Principle):** The meaning of a complex expression is determined by the meanings of its parts and the way they are syntactically combined.
- Tree diagrams

(1) **Phrase structure rules**
$S \rightarrow$ not S
$S \rightarrow$ DP VP
$VP \rightarrow V_{itr}$
$VP \rightarrow V_{tr}$ DP
$VP \rightarrow$ is AdjP
$VP \rightarrow$ is DP
$DP \rightarrow D\ NP$
$DP \rightarrow$ ProperN
$AdjP \rightarrow Adj$
...

(2) **Vocabulary**
$V_{itr} \rightarrow$ meow, bark, ...
$V_{tr} \rightarrow$ invite, ...
ProperN $\rightarrow$ Kitty, ...
...

(3)



- Basic composition rules

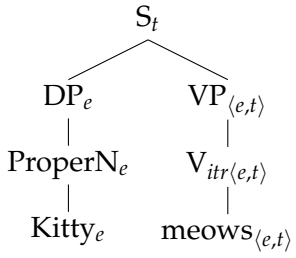
(4) **Terminal Nodes (TN)**
If α is a terminal node, $[\![\alpha]\!]$ is specified in the lexicon.

Non-Branching Nodes (NN)
If α is non-branching node, and β is its daughter node, then $[\![\alpha]\!] = [\![\beta]\!]$.

Functional Application (FA)
If $\{\beta, \gamma\}$ is the set of α's daughters, $[\![\beta]\!] \in D_{\langle\sigma,\tau\rangle}$, and $[\![\gamma]\!] \in D_{\sigma}$, then $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$

1.2. Composition with intransitive verbs

- Tree diagram



Applying composition rules:

$[\![S]\!]^w = [\![VP]\!]^w([\![DP]\!]^w)$ By FA

$= [\![V_{itr}]\!]^w([\![ProperN]\!]^w)$ By NN

$= [\![meows]\!]^w([\![Kitty]\!]^w)$ By TN

- Inserting lexicons
  - If we think of predicates as denoting sets of entities, then the composition of "Kitty" and "meows" proceeds via **set membership**:

(5) $[\![\text{meows}]\!]^w([\![\text{Kitty}]\!]^w) = 1$ iff $[\![\text{Kitty}]\!]^w \in [\![\text{meows}]\!]^w$
iff $\textsf{Kitty} \in \{x : x \text{ meows in } w\}$

– If we think of predicates as denoting characteristic functions (from entities to truth values), then the composition of "Kitty" and "meows" proceeds via **functional application**:

(6) $[\![\text{meows}]\!]^w([\![\text{Kitty}]\!]^w) = \left[\begin{array}{l} f : D_e \to D_t \\ \quad \text{for all } x \in D_e,\ f(x) = 1 \text{ iff } x \text{ meows in } w \end{array}\right](\textsf{Kitty})$
$= 1$ iff Kitty meows in w.

1.3. Compose with transitive verbs

- Recall the rules for predications in predicate logic:

(7) a. Syntax
i. If P is a one-place predicate, and a is a term, then $P(a)$ is a wff.
ii. If P is a two-place predicate, and a and b are terms, then $P(a, b)$ is a wff.
b. Semantics
i. $[\![P(a)]\!]^{M,g} = 1$ iff $[\![a]\!]^{M,g} \in [\![P]\!]^{M,g}$.
ii. $[\![P(a, b)]\!]^{M,g} = 1$ iff $\langle [\![a]\!]^{M,g}, [\![b]\!]^{M,g} \rangle \in [\![P]\!]^{M,g}$.

- Now, think about the composition of a sentence with a transitive predicate.

(8) Andy invited Billy.

- **Attempt 1:**

Previously, we defined a transitive verb as a two-ary relation (viz., a set of pairs of individuals), as in (9a), or a characteristic function from pairs of individuals to truth values, as in (9b-c).

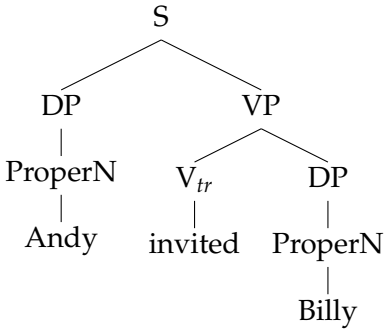
(9) a. $[\![\text{invited}]\!]^w = \{\langle x, y\rangle \mid x \text{ invited } y \text{ in } w\}$
b. $[\![\text{invited}]\!]^w = f : D_{(e,e)} \to D_t$ such that
for every individual $x \in D_e$ and $y \in D_e$, $f(x, y) = 1$ iff x invited y in w.
c. (w: Andy invited Billy, and nobody else invited anyone.)
$$[\![\text{invited}]\!]^w = \left[\begin{array}{lcl} \langle a, b\rangle & \to & 1 \\ \langle a, a\rangle & \to & 0 \\ \langle b, a\rangle & \to & 0 \\ \langle b, b\rangle & \to & 0 \end{array}\right]$$

This lexicon and the above predication rule require the transitive verb *invited* to be combined with a pair of individuals.

(10) a. *In terms of set membership*
$[\![\text{Andy invited Billy}]\!]^w = 1$ iff $\langle [\![\text{Andy}]\!]^w, [\![\text{Billy}]\!]^w \rangle \in [\![\text{invited}]\!]^w$

b. *In terms of functional application*
$[\![\text{Andy invited Billy}]\!]^w$
$= [\![\text{invited}]\!]^w([\![\text{Andy}]\!]^w, [\![\text{Billy}]\!]^w)$
$= \left[\begin{array}{l} f : D_{(e,e)} \to D_t \\ \quad \text{for all } \langle x, y\rangle \in D_{(e,e)},\ f(x, y) = 1 \text{ iff } x \text{ invited } y \text{ in } w \end{array}\right](\textsf{Andy}, \textsf{Billy})$
$= 1$ iff Andy invited Billy in w.

Problem: However, consider the tree diagram, what is the meaning of VP? Note that in the tree diagram, the transitive verb combines with the object DP, not directly with a pair.



- **Attempt 2:**

  Lexicon of a transitive predicate: we convert any n-ary relation or (a characteristic function for n-tuples) into an n-place function.

  (11) a. $[\![\text{invited}]\!]^w = f : D_e \to D_{\langle e,t\rangle}$ such that
  for all $y \in D_e$, $f(y) = g : D_e \to D_t$ such that
  for all $x \in D_e$, $g(x) = 1$ iff x invited y in w.

  b. (w: Andy invited Billy, and nobody else invited anyone.)

$$[\![\text{invited}]\!]^w = \begin{bmatrix} b & \to & \begin{bmatrix} a & \to & 1 \\ b & \to & 0 \end{bmatrix} \\ a & \to & \begin{bmatrix} a & \to & 0 \\ b & \to & 0 \end{bmatrix} \end{bmatrix}$$

Object $\to$ [subject $\to$ truth value]

Now compose VP:

(12)
$$\begin{aligned} [\![\text{VP}]\!]^w &= [\![\text{V}_{tr}]\!]^w([\![\text{DP}]\!]^w) && \text{By FA} \\ &= [\![\text{V}_{tr}]\!]^w([\![\text{ProperN}]\!]^w) && \text{By NN} \\ &= [\![\text{invited}]\!]^w([\![\text{Billy}]\!]^w) && \text{By TN} \\ &= \left[\begin{array}{l} f : D_e \to D_{\langle e,t\rangle} \\ \quad \text{for all } y \in D_e,\ f(y) = g : D_e \to D_t \\ \qquad\qquad \text{for all } x \in D_e,\ g(x) = 1 \text{ iff } x \text{ invited } y \text{ in } w \end{array}\right] (\textsf{Billy}) \\ &= \left[\begin{array}{l} g : D_e \to D_t \\ \quad \text{for all } x \in D_e,\ g(x) = 1 \text{ iff } x \text{ invited Billy in } w \end{array}\right] \end{aligned}$$

- More generally, let's revise the rules for predications to the following:

  (13) a. Syntax
  i. If P is a one-place predicate, and a is a term, then $P(a)$ is a wff.
  ii. If P is a two-place predicate, and a is a term, then $P(a)$ is a one-place predicate.

  b. Semantics
  If P is a one-place predicate, and a is a term, then $[\![P(a)]\!]^{M,g} = [\![P]\!]^{M,g}([\![a]\!]^{M,g})$

1.4. Extension: Sentential connectives

- Recall the interpretation rules in propositional logic

(14) a. **Negation**
$[\![\neg p]\!]^w = 1$ iff $[\![p]\!]^w = 0$.

b. **Binary connectives**
$[\![p \wedge q]\!]^w = 1$ iff $[\![p]\!]^w = 1$ and $[\![q]\!]^w = 1$.
$[\![p \vee q]\!]^w = 1$ iff $[\![p]\!]^w = 1$ or $[\![q]\!]^w = 1$.
$[\![p \rightarrow q]\!]^w = 1$ iff $[\![p]\!]^w = 0$ or $[\![q]\!]^w = 1$.
$[\![p \leftrightarrow q]\!]^w = 1$ iff $[\![p]\!]^w = [\![q]\!]^w$.

These rules allow us to interpret sentences containing sentential connectives. Now let's see how to define sentential connectives and to derive the meaning of these sentences compositionally. (For now, we assume that sentential connectives combine with the extensional values of sentences.)

- Analogous to the case of an intransitive verb, we can define negation as a characteristic function from truth values to the opposite truth values.

(15) Negation *not*

a. $[\![\text{not}]\!]^w = f : D_t \rightarrow D_t$ such that for every $v \in D_t$, $f(v) = 1$ iff $v = 0$

b. $[\![\text{not}]\!]^w = \begin{bmatrix} 1 & \rightarrow & 0 \\ 0 & \rightarrow & 1 \end{bmatrix}$

NB: We shall distinguish between lexical items which have a **world-dependent** semantic value and those that are **world-independent**. Predicates (including Vs, CNs, predicative Adjs, relative clauses) are typically world-dependent. Truth-functional connectives, determiners, and proper names have semantic values which do not differ from worlds to worlds.

- Analogous to the case of a transitive verb, we can define a binary connective as a 2-ary relation, a characteristic function, or a 2-place function.

(16) Conjunctive *and*

a. As a 2-ary relation $\quad [\![\text{and}]\!]^w = \{\langle 1,1\rangle\}$

b. As a characteristic function

i. $[\![\text{and}]\!]^w = f : D_{(t,t)} \rightarrow D_t$ such that
for every $v_1 \in D_t$ and $v_2 \in D_t$, $f(v_1, v_2) = 1$ iff $v_1 = v_2 = 1$

ii. $[\![\text{and}]\!]^w = \begin{bmatrix} \langle 1,1\rangle & \rightarrow & 1 \\ \langle 1,0\rangle & \rightarrow & 0 \\ \langle 0,1\rangle & \rightarrow & 0 \\ \langle 0,0\rangle & \rightarrow & 0 \end{bmatrix}$

c. As a 2-place function

i. $[\![\text{and}]\!]^w = f : D_t \rightarrow D_{\langle t,t\rangle}$ such that
for every $v_2 \in D_t$, $f(v_2) = g : D_t \rightarrow D_t$ such that
for every $v_1 \in D_t$, $g(v_1) = 1$ iff $v_1 = v_2 = 1$

ii. $[\![\text{and}]\!]^w = \begin{bmatrix} 1 & \rightarrow & \begin{bmatrix} 1 & \rightarrow & 1 \\ 0 & \rightarrow & 0 \end{bmatrix} \\ 0 & \rightarrow & \begin{bmatrix} 1 & \rightarrow & 0 \\ 0 & \rightarrow & 0 \end{bmatrix} \end{bmatrix}$

Exercise: Define *or* as a 2-ary relation, a characteristic function, and a 2-place function.