Type theory

1. Type theory

- Types are simply sets of objects. For example, in math, the type N is the set of natural numbers. In semantics, we work with two basic types which correspond to the categories that Frege takes to be "saturated", and derive complex types recursively based on these two types:[1]

  (1) **Types**
  - a. **Basic types**: e (for individuals/entities) and t (for truth values).
  - b. **Functional types**: If α and β are types, then $\langle \alpha, \beta \rangle$ is a type. A function of type $\langle \alpha, \beta \rangle$ is one whose arguments/inputs are of type α and whose values/outputs are of type β.

  (2) **Domains**
  - a. $D_t = \{1, 0\}$
  - b. $D_e = \{x \mid x \text{ is an entity}\}$
  - c. $D_{\langle \alpha,\beta \rangle} = \{f \mid f : D_\alpha \rightarrow D_\beta\}$ (functions from things of type α to things of type β)

- Adding possible worlds to theory of types:[2]

  (3) a. **Intensional types**: If σ is a type, then $\langle s, \sigma \rangle$ is an intensional type.
  b. $D_{\langle s,\tau \rangle} = \{f \mid f : W \rightarrow D_\tau\}$ (functions from possible worlds to things of type τ)

  In this class, we will mostly work with extensional types.

- Syntactic categories and their semantic types (an inclusive list)

Syntactic category	**Category label**	**English expressions**	**Semantic type** (extensionalized)
Sentence	S		t
Proper name	ProperN	*John*	e
e-type/referential DP	DP	*the king*	e
Common noun	CN	*cat*	$\langle e, t \rangle$
Intransitive verb, VP	V_{itr}, VP	*run*	$\langle e, t \rangle$
Transitive verb	V_{tr}	*love*, *buy*	$\langle e, et \rangle$
Predicative adjective	Adj	*happy*, *gray*	$\langle e, t \rangle$
Predicate modifier	Adv	*skillful*, *quickly*	$\langle et, et \rangle$
Determiner	D	*some*, *every*, *no*	$\langle et, \langle et, t \rangle \rangle$
	D	*the*	$\langle et, e \rangle$

[1] Types in semantics have one-to-one correspondence to syntactic categories in Frege's category grammar. Frege assumes that linguistic communication involves two saturated categories: NAME (N) which picks out some entity in the world, and SENTENCE (S) which says something about that entity. Their semantic counterparts are type e and type t, respectively. Other categories are derived out of these two categories. For example, an intransitive verb is of category S/N and of type $\langle e, t \rangle$.

[2] Note that we are not actually adding s for possible worlds to our type theory. This is because (as far as we've seen) there are no expressions of natural language that have specific possible worlds as their values.

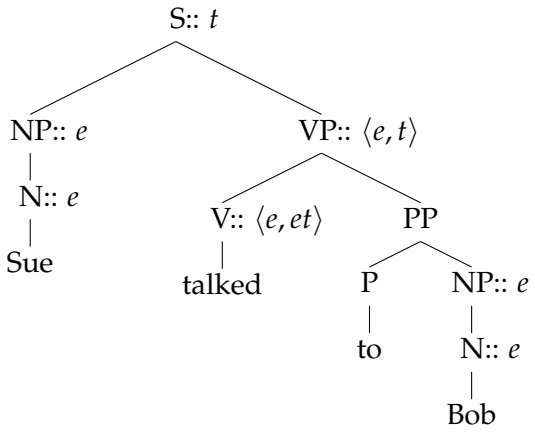
Exercise: Identify the semantic type of *beautiful* in the following sentences:

(4) a. Jenny is a *beautiful* girl.
b. Jenny is a *beautiful* dancer.

Exercise: Classify the following words based on their semantic types:
not, if...then, student, John, Boston, a man, buy, fast, carefully, necessarily

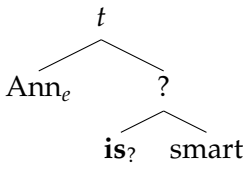
- In most cases, with type assignments to expressions of natural language, we can determine the semantic types of new expressions/morphemes.

(5) Susan talked to Bob.

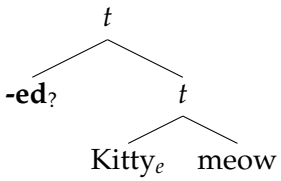


Exercise: Identify the semantic types of the underlined words/morphemes.

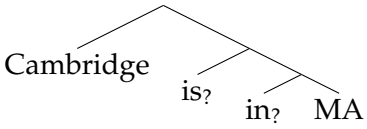
(6) Ann is smart.



(7) Kitty meowed.



(8) Cambridge is in MA.



- But, be careful!
  - some expressions can be type-ambiguous;
  - there can be covert elements in the LF;
  - there can be type-shifting operations;
  - two sister nodes might not hold a function-argument relation; ...